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13. ABSTRACT *(Maximum 200 words)*

This publication described and illustrates the application of statistics in hydrologic engineering, especially flood frequency analysis. The subject matter covers the following items:

(1) A review of the basic concepts of probability and correlation analyses that is applicable in hydrologic engineering, with a guide to supplemental reading.

(2) Presentation of detailed computation procedures and computation aids for derivation of frequency estimates based on analysis of hydrologic record.

(3) A summary of procedures for developing "regionalized" hydrologic frequency estimates, based on analyses of hydrologic records available at stream gaging stations, adjusted to provide generalized flood-frequency relations that are considered to be representative of long-period hydrologic characteristics.

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US Army Corps of Engineers
Institute for Water Resources
Hydrologic Engineering Center
609 Second Street
Davis, CA 95616

(530) 756-1104
(530) 756-8250 FAX
www.hec.usace.army.mil

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FOREWORD

Since publication of the original paper of this title in July 1952, the general concept of runoff frequency analysis contained in the paper has been used and developed extensively by the Corps of Engineers and other organizations. It is considered appropriate at this time to expand the paper to include the new developments and associated computation techniques and examples.

While there are yet many difficulties encountered in application, the method of frequency analysis originally proposed (logarithmic Pearson Type III) has been generally very successful in the accurate and rapid determination of extreme flood frequencies. It is considered adequate for all hydrologic frequency applications, and consequently, the treatment herein is still restricted to the originally proposed method. In the interest of simplicity, terms and concepts that are not strictly necessary to this method are not discussed, even though they may be in common use in hydrologic statistics.

So many have contributed toward the material contained herein that it is impossible to acknowledge even the principal contributors. However, most of the newer developments originated under the Civil Works Investigations program of the Corps of Engineers, particularly the project conducted in the Sacramento District under the direction of the writer and under the general administration of Mr. F. Kochis, Chief of the Engineering Division, and Mr. A. Gomez, Chief of the Planning and Reports Branch. Mr. A. L. Cochran in the Office, Chief of Engineers, has guided the program and has provided invaluable support and constant encouragement.

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STATISTICAL METHODS IN HYDROLOGY

SECTION 1 - INTRODUCTION

1-01. NATURE AND OBJECTIVES OF STATISTICAL ANALYSIS

Statistical analysis as applied in hydrologic engineering consists of (a) estimating the future frequency or probability of hydrologic events based on information contained in hydrologic records and (b) correlating interrelated hydrologic variables. In probability analyses, statistical methods permit coordination of observed data to yield a more accurate estimate of future frequencies than is indicated by the raw data, and also provide criteria for judging the reliability of the frequency estimates. In correlation analyses, statistical methods provide means for deriving the most likely relationship between two variables, and also provide criteria for judging the reliability of forecasts or estimates based on the derived relationship.

1-02. PURPOSE AND SCOPE

a. This is a revision of the paper of the same title dated July 1952 and distributed to Corps of Engineers offices by Engineer Bulletin 52-24, dated 26 August 1952. The revision incorporates new material developed under the Civil Works Investigations program of the Corps.

b. It is the purpose of this publication to describe and illustrate the application of statistics in hydrologic engineering. The subject matter covers the following items:

(1) A concise review of the basic concepts of probability and correlation analyses that are applicable in hydrologic engineering, with a guide to supplemental reading for further treatment.

(2) Presentation of detailed computation procedures and supporting justifications and computation aids for derivation of probability of frequency estimates based on analysis of hydrologic records that have been adjusted as required to conform with selected reference base conditions.

(3) A summary of procedures for developing "regionalized" hydrologic frequency estimates, based on analyses of hydrologic records available at stream gaging stations, adjusted to provide generalized flood-frequency relations that are considered most representative of long-period hydrologic characteristics in various drainage areas in the region. Also, illustrations and explanations

of simple generalization procedures for use where these are adequate and advantageous are given.

(4) A brief list of key questions and simple problems suitable for use in training classes. More detailed training documents have been prepared in connection with training courses given under Civil Works Investigation Project CW-151 in the Sacramento District of the Corps of Engineers.

(5) Discussions pertaining to certain aspects of statistical analyses associated with hydrologic engineering that deserve special emphasis.

c. For those who are interested only in frequency analyses of flood discharges, sections 3 and 4 and exhibits 2 to 4 contain the essential guide material. For those interested only in correlation analyses, section 9 contains the essential material. The detailed procedure for computing the frequencies of flood peaks and volumes is given step by step on exhibit 18.

1-03. REFERENCES

There are a great many textbooks on statistics, probability, and correlation, and a multitude of technical papers on the statistical aspects of hydrologic engineering. A few references considered particularly applicable and useful for the purpose of supplementing material contained herein are given at the end of the text.

1-04. ORGANIZATION OF THIS PAPER

The technical presentation in this paper begins with a brief discussion of statistical concepts and definitions contained in section 2. Graphical methods of frequency analyses are covered in section 3, and numerical or analytical procedures are contained in section 4. A recommended procedure for adjusting frequency estimates based on short records, using data at long record stations, is contained in section 5. Extension of the methods discussed in section 4 for use in estimating frequencies of flood volumes is contained in section 6. Recommended means for coordinating frequency estimates within a region for the purposes of increasing the reliability of estimates and for deriving estimates for ungaged areas are contained in section 7. A brief description of the application of frequency procedures to hydrologic factors other than runoff is contained in section 8. Section 9 contains a brief exposition of correlation methods generally in use. These are readily available in many textbooks, but they are given here for convenience and for the purpose of standardizing notation. Criteria for evaluating the reliability of frequency statistics, frequency estimates and correlation results are contained in section 10. Terms and symbols as used herein are summarized in section 11. Following this is given a list of selected references. Common questions and answers intended to clarify some of the more complex aspects of probability analysis are contained in Appendix I, and illustrative problems for use as exercises in connection with a frequency course are given in Appendix II.

SECTION 2 - GENERAL PROBABILITY CONCEPTS AND DEFINITIONS

2-01. INTRODUCTION

The subjects of probability and statistics are becoming increasingly applicable in engineering work, and it is considered appropriate to provide background information for orienting those engineers who have not had formal training in the subjects. This section contains a brief review of the theoretical basis for probability estimates based on observed data. Details of application will be presented later, and a broader understanding of the theory can be obtained from textbooks such as reference 1.

2-02. NATURE OF RANDOM EVENTS

a. Probability estimates made in hydrologic engineering are based on records of random events. To understand probability methods and fully appreciate the degree of reliability of such probability estimates, one should consider the nature and variation of random samples.

b. Consider a period of 2,000 years during which controlling hydrologic conditions do not change. Annual maximum hydrologic events occurring during this period can be divided into 100 records of 20 years each. From knowledge of probability, it is expected that one of these records will contain a flood that is exceeded on the average only once in 2,000 years, a very rare event. About 18 of these records will contain floods that are exceeded on the average only once in 100 years (it would be 20, except that some of the records might contain more than one of these large floods), and 64 of the records should contain floods larger than that exceeded on the average once in 20 years. On the other hand, about 12 of the records would not have floods larger than that exceeded on the average once in 10 years.

c. When a hydrologic engineer is studying a record of 20 years' length, he cannot tell by examining the record alone whether it is one that has a normal sequence of events, abnormally rare events, or an abnormally small number of large events. If the record contains abnormally large events, the resulting probability estimates for large events will be too high, and vice versa. In order to reduce the uncertainties from this source, it is advisable to study all of the events in relation to each other and to introduce knowledge obtained on similar phenomena at other locations.

2-03. PROBABILITY INFERENCE

a. Knowledge that a certain set of conditions can result in various sets of data because of random variations is used to infer that any particular set of data could have resulted from various sets of hydrologic conditions. The problem in making probability estimates is essentially to determine the set of conditions that most likely generated the sample of data that has been recorded. This set of conditions is represented by a "parent population" which consists of all of the hydrologic events that would be generated if the record continues indefinitely and controlling conditions do not change.

b. In probability analysis, there are two basic approaches to estimating or inferring the parent population from sample data. First, data can be arranged in the order of magnitude to form a frequency array, illustrated on exhibit 1, and a graph of magnitude versus observed frequency plotted, as shown on exhibit 2. A smooth curve drawn through the plotted data would represent an estimate of the parent population from which the probabilities of future events can be determined. The second basic approach is to derive from the data general statistics representing the average magnitude of floods, the variability from that average, and any other pertinent statistics relating frequencey to magnitude as indicated by the data.

2-04. CUMULATIVE FREQUENCY CURVES

a. A cumulative frequency curve, or simply frequency curve, such as that illustrated on exhibit 2, relates the magnitude of an event to the frequency with which that magnitude is exceeded as events occur at random. For example, if 25 floods at a location exceed 10,000 c.f.s. in 100 years, on the average, then the value of 10,000 c.f.s. on the cumulative frequency curve will correspond to an exceedence probability of 0.25 in any 1 year or an exceedence frequency of 25 times per 100 years, 250 times per thousand years, etc., or simply 25 percent. While a single frequency curve can represent the frequency of peak discharges at a given location, the frequency of flood volumes would be represented by an entirely different curve, or by more than one curve if volumes for various durations are concerned. Frequency curves can also be used to represent the frequency of reservoir stages, river stages, precipitation, and many other phenomena.

b. Frequency curves are most commonly used in flood control benefits studies for the purpose of evaluating the economic effect of the project. Other common uses of frequency curves include the determination of reservoir stage frequency for real estate acquisition and reservoir-use purposes, the selection of rainfall frequency for storm-drain design, and the selection of runoff frequencies for interior drainage, pumping plant, and local-protection project design.

c. The basic frequency curve used in hydrologic engineering is the frequency curve of annual maximum or annual minimum events. A second curve, the partial-duration curve, represents the frequency of all events above a given base value, regardless of whether two or more occurred in the same year. Either curve must be supplemented by considerations of seasonal effects and other factors in application, as explained in reference 5. When both the frequency curve of annual floods and the partial-duration curve are prepared, care must be exercised to assure that the two are consistent. Normal relationships between the two are given in paragraph 4-04.

d. In almost all locations there are seasons during which storms or floods do not occur or are not severe, and other seasons when they are more severe. Also, damages associated with a flood often vary with season of the year, among other factors. In many types of studies, the seasonal variation factor is of primary importance, and it becomes necessary to establish frequency curves for each month or other subdivision of the year. For example, one frequency curve might represent the largest floods that occur each January, a second one would represent the largest floods that occur each February, etc. In another case, one frequency curve might represent floods during the snowmelt season, while a second might represent floods during the rain season. Occasionally, when seasons are studied separately, an annual-event curve covering all seasons is also prepared, and care should be exercised to assure that the various seasonal curves are consistent with the annual curve (reference 19).

e. In connection with power studies for run-of-river plants particularly, and in some phases of sediment studies, the flow-duration curve serves a useful purpose. It simply represents the percent of time during which specified flow rates are exceeded at a given location. Ordinarily, variations within periods less than one day are not of consequence, and the curves are therefore based on observed mean-daily flows. For the purpose served by flow-duration-curves, the extreme rates of flow are not important, and consequently there is no need for refining the curve in regions of high flow. The procedure ordinarily used in the preparation of a flow-duration curve consists of counting the number of mean-daily flows that occur within given ranges of magnitude. Then the lower limit of magnitude in each range is plotted against the percentage of days of record that mean-daily flows exceed that magnitude. A typical flow-duration curve is shown on exhibit 9.

2-05. TECHNICAL APPROACH IN ESTIMATING FREQUENCY CURVES

a. There are two basic approaches to estimating frequency curves--graphical and analytical. Each of these approaches has several variations in current practice, but the discussion herein will be limited to selected methods.

b. Graphically, frequencies are evaluated simply by arranging observed values in the order of magnitude and considering that a smooth curve suggested by that array of values is representative of future possibilities. Each value represents a fraction of the future possibilities and, when plotting the frequency curve, it is given a "plotting position" that is calculated to give it the proper weight (see paragraph 3-05).

c. In the application of analytical (statistical) procedures, the concept of theoretical populations or distributions is employed, as discussed in paragraph 2-03. A distribution is a set of values that would occur under fixed conditions in an infinite amount of time. Those that have occurred are presumed to constitute a random sample and accordingly are used to make particular inferences regarding their "parent population" (i.e., the distribution from which they were derived). Such inferences are necessarily attended by considerable uncertainty, because a given set of observations could result from any of many sets of physical conditions (from any one of many distributions). However, by the use of statistical processes, the most probable nature of the distribution from which the data were derived can be estimated. Since in all probability this is not the true parent population, the relative chance that variations from this "maximum likelihood" distribution might be true must be evaluated. Each range of possible parent population is then weighted in proportion to its likelihood to obtain a weighted average as demonstrated in reference 4. A probability obtained from this weighted average is herein referred to as the expected probability, P_N. Computation procedures are given in section 4.

d. Because of the shortness of hydrologic records, frequency determinations are relatively unreliable where based on a single record (see paragraph 10-03). Also, it is often necessary to estimate frequencies for locations where no record exists. For these reasons, regionalized frequency studies, in which frequency characteristics are related to drainage-basin features, are desirable. These are facilitated by the use of analytical methods, as illustrated in section 7.

2-06. TERMS AND SYMBOLS

Special terms and symbols are defined where first used herein. A summary is given in section 11 for convenience.

SECTION 3 - FLOOD PEAK FREQUENCY - GRAPHICAL METHOD

3-01. USE AND RELATIVE ADVANTAGES

Every frequency study should be plotted graphically, even though the results can be obtained entirely analytically as described in section 4, in order that observed data may be visually compared with the derived curve. The graphical method of frequency-curve determination can be used for any type of frequency study, but analytical methods have certain advantages where they are applicable (see paragraph 4-01). The principal advantages of graphical methods are that they are generally applicable, that the derived curve can be easily visualized, and that the observed data can be readily compared with the computed results. However, graphical methods of frequency analysis are inferior in accuracy to analytical methods where the latter apply, and do not provide means of evaluating the reliability of the estimates. Comparison of the adopted curve with plotted points is not an index of reliability as in correlation analysis, but it is often erroneously assumed to be, thus implying a much greater reliability than is actually attained. For these reasons, graphical methods should be limited to those cases where analytical methods do not apply (that is, where frequency curves are too irregular to compute analytically) and to use as a visual aid or check on analytical computations.

3-02. GENERAL PROCEDURE

a. Graphical construction of a frequency curve simply consists of arranging the selected data in the order of magnitude and plotting the magnitude of each item on the vertical scale against its estimated exceedence frequency (plotting position - see paragraph 3-05) on the horizontal scale, using a suitable grid (see paragraph 3-06). A smooth curve drawn through the points is the desired frequency curve.

b. Data used in the construction of frequency curves of peak flows consist of the maximum flow for each year of record and all of the secondary flows that exceed a selected base value. This base value must be smaller than any floodflow that is of importance in the analysis, and should also be low enough so that the total number of floods in excess of the base equals or exceeds the number of years of record. Ordinarily, the latter criterion controls, and the two series of events tabulated are equal in number. Exhibit 3 is a sample tabulation of data directly from the record, of the frequency arrays in the order of magnitude, and of corresponding plotting positions. In arranging the data in order of magnitude, much time can be saved by taking the events in the order in which they occurred, and placing them rapidly on a blank sheet of paper in the order of magnitude, thus making an irregular tabulation. This tabulation is then recopied onto the form shown on exhibit 3 and cross-checked with chronologic values.

3-03. SELECTION AND ARRANGEMENT OF DATA

a. The primary consideration in selection of an array of data for a frequency study is the use to which the frequency estimates will be put. If the frequency curve is to be used for estimating damages that are related to instantaneous peak flows in a stream, peak flows should be selected from the record. If the damages are related to maximum mean-daily flows or to maximum 3-day flows, these items should be selected. If the behavior of a reservoir under investigation is related to the 3-day or 10-day rainflood volume, or to the seasonal snowmelt volume, that pertinent item should be selected. Occasionally, it is necessary to select a related variable in lieu of the one desired. For example, where mean-daily flow records are more complete than the records of peak flows, it may be more desirable to derive a frequency curve of mean-daily flows and then, from the computed curve, derive a peak-flow curve by means of an empirical relation between mean-daily flows and peak flows. All reasonably independent values should be selected, but the annual maximum events should ordinarily be segregated when the application of analytical procedures discussed in section 4 is contemplated.

b. Data selected for a frequency study must measure the same aspect of each event (such as peak flow, mean-daily flow, or flood volume for a specified duration), and each event must be controlled by a uniform set of hydrologic and operational factors. For example, it would be improper to combine items from old records that are reported as peak flows but are in fact only daily readings, with newer records where the peak was actually measured. Similarly, care should be exercised when there has been significant change in upstream storage regulation during the period of record so as not inadvertently to combine unlike events into a single series. In such a case, the entire record should be adjusted to a standard condition.

c. Hydrologic factors and relationships operating during a general winter rainflood are usually quite different from those operating during a spring snowmelt flood or during a local summer cloudburst flood. Where two or more types of floods are distinct and do not occur predominantly in mutual combinations, they should not be combined into a single series for frequency analysis. It is usually more reliable in such cases to segregate the data in accordance with type and to combine only the final curves, if necessary. In the Sierra Nevada region of California and Nevada, frequency studies are made separately for rainfloods, which occur principally during the months of November through March, and for snowmelt floods, which occur during the months of April through July. Flows for each of these two seasons are segregated strictly by cause - those predominantly caused by snowmelt and those predominantly caused by rain. In desert regions, summer thunderstorms should be excluded from frequency studies of winter rainfloods or spring snowmelt floods and should be considered separately.

d. Occasionally a runoff record may be interrupted by a period of one or more years. If the interruption is caused by destruction of the gaging station by a large flood, failure to fill in the record for that flood would have a biasing effect, which should be avoided. However, if the cause of the interruption is known to be independent of flow magnitude, the entire period of interruption should be eliminated from the frequency array, since no bias would result. Knowledge we have about floods observed at other locations during periods of no record at the site concerned can be utilized as discussed in section 5. In cases where no runoff records are available on the stream concerned, it is usually best to estimate the frequency curve as a whole using regional generalizations discussed in section 7, instead of attempting to estimate a complete series of individual floods, because ordinary methods of estimating individual floods tend to reduce the slope of the frequency curve (the standard deviation).

3-04. ADJUSTMENT TO UNIFORM CONDITION

Since the frequency analysis of hydrologic data is based on the assumption of random occurrences, each item of data must have occurred under similar hydrologic conditions or must be adjusted to a standard uniform condition. If control by reservoirs or diversion for irrigation, etc., has affected the runoff, some adjustment of the data is usually necessary. Where it is feasible to adjust to natural conditions, it is advisable to do so in order that the data will more nearly conform to theoretical frequency functions that have been found to describe the frequency of natural hydrologic events. This is accomplished by standard routing procedures, and in many cases an approximate adjustment is satisfactory. Where the regulation is complex, as in the case of a large number of upstream reservoirs, it may be advisable to adjust the data to a uniform condition with specific reservoirs and diversion facilities operating. For design purposes, a frequency curve of runoff under "non-project" conditions that is expected to prevail during the lifetime of the proposed project, if the project is not constructed, is required. A frequency curve based on any specified uniform condition can be converted to one for nonproject conditions using relationships developed by routing "balanced" floods of specified frequency (i.e., floods having runoff for various durations and in various portions of the drainage basin of equal exceedence frequency). Techniques for doing this are outside the scope of this paper.

3-05. PLOTTING FORMULA

a. The reasoning behind the selection of an exact formula for plotting the frequency of observed flood events is extremely complex. Approximate plotting positions can be obtained by reasoning that each item in a set of, say 10, represents 10 percent of the parent-population events and should be plotted in the middle of its group, that is at 5, 15, 25 percent, etc., for successive events in the order of magnitude. The formula derived from this line of reasoning has been used in the past, and is generally satisfactory, considering the overall reliability of the results. However, more accurate plotting positions have been derived theoretically and, since their use is very simple, it is considered advantageous to use them. Plotting positions recommended are shown on exhibit 37, and are based on the premise that if they are used repeatedly in a great number of random samples, they will prove to be too low in half of the cases and too high in the other half, compared with the theoretically true values that cannot be determined because of random variations in data. They are, therefore, called median plotting positions. There are other systems of deriving plotting positions that yield good results, but of the formulas generally used, use of the median plotting position will most nearly duplicate results obtained by analytical methods of frequency analysis that do not require plotting positions.

b. In ordinary hydrologic frequency work, exceedence frequencies are expressed in percent or in terms of events per hundred years, as shown on exhibit 37, which give plotting positions for arrays up to 100 events in size. For arrays larger than 100, the plotting position, P, can be obtained as were those of exhibit 37 by use of the following equation:

$$1 - P_1 = (0.5)^{1/N} \qquad (1)$$

in which P_1 is the plotting position for the largest event, and N is the number of years of record. The plotting position for the smallest event is the complement of this value, and all other plotting positions are interpolated linearly between these two. For partial-duration curves, particularly where there are more events than years (N), plotting positions larger than 50 percent are obtained by use of the following equation:

$$P = (2m - 1)/2N \qquad (1a)$$

in which m is the order number of the event.

3-06. PLOTTING GRID

If hydrologic frequency data are plotted with Cartesian coordinates, the resulting frequency relationship will curve rather abruptly at the upper end and possibly at the lower end also. Furthermore, the extreme values in which there is the greatest interest would be compressed into a very small area, and extrapolation of the curve would be difficult. Accordingly, it has been found desirable to use a plotting grid on which a frequency curve of hydrologic data will usually approximate a straight line. A grid that has been found to be suitable for this purpose is the probability grid. This grid is designed so that the cumulative frequency curve of a variable that is distributed in accordance with the normal probability curve will plot as a straight line. The grid is illustrated on exhibit 10. It has been found that items such as air temperature or river stage that either do not have a fixed lower limit of zero or whose lower limit is far removed from the range of experience, will often yield frequency curves approximating a straight line when plotted on this grid. Variables such as streamflow where a lower limit of zero is often approached in experience will ordinarily yield an approximately straight frequency curve only if the logarithms are plotted on this grid. For convenience, the logarithmic probability grid illustrated on exhibit 2 has been devised so that flows can be plotted directly to yield an approximately straight line.

3-07. SAMPLE COMPUTATION

a. Exhibit 2 shows the plotting of a frequency curve of annual peak flows corresponding to those tabulated on exhibit 3. The curve should be drawn so as to balance out the plotted points to a reasonable degree and so that there is no abrupt break in the frequency curve. Unless computed as discussed in section 4, the frequency curve should be drawn as a straight line on the grid whenever data reasonably indicate a straight line and conditions do not exist that would make a straight line unreasonable.

b. The partial-duration curve corresponding to the partial-duration data on exhibit 3 has been shown on exhibit 8. This curve has been drawn by generally balancing out the plotted points except that it was made to conform with the annual-event curve in the upper portion in general accord with the standard relationship discussed in paragraph 4-04. When partial-duration data must include more events than there are years of record (see paragraph 3-02) it will be necessary to use logarithmic paper for plotting purposes, as on exhibit 8, in order to plot exceedence frequencies greater than 100 percent. Otherwise, the curve can be plotted on probability grid, as illustrated on exhibit 20.

c. Exhibit 10 illustrates the graphical construction of a river-stage frequency curve. The shape of this curve is dictated by the plotted points and, in some cases, by consideration of the stage at which overbank flows begin. Whenever stage is a consistent function of flow, as is the usual case, the stage-frequency curve should be obtained from the flow-frequency and stage-discharge curves.

SECTION 4 - FLOOD PEAK FREQUENCY - ANALYTICAL COMPUTATION

4-01. USE, LIMITATIONS AND RELATIVE ADVANTAGES

The analytical method of computing a frequency curve in hydrologic engineering is limited almost exclusively to curves of annual maximum or annual minimum streamflows for a specified duration (including peak flows) and annual maximum precipitation amounts for a specified duration. In general, the results obtained by analytical methods are considerably more reliable than those obtained by graphical procedures. They have the additional advantages that the degree of reliability of frequency estimates can be evaluated, as discussed in paragraph 10-03.

4-02. EQUATIONS USED

a. Frequency curves are computed analytically by the use of moments of the logarithms, expressed in terms of the mean, M, (first moment), standard deviation, S, (second moment) and skew coefficient, g, (third-moment function). The three corresponding equations used are as follows:

$$M = \Sigma X/N \qquad (2)$$

$$S^2 = \frac{\Sigma x^2}{N-1} = \frac{\Sigma X^2 - (\Sigma X)^2/N}{N-1} \qquad (3)$$

$$g = \frac{N \quad \Sigma x^3}{(N-1)(N-2)S^3} = \frac{N^2\Sigma X^3 - 3N\ \Sigma X\ \Sigma X^2 + 2(\Sigma X)^3}{N(N-1)(N-2)S^3} \qquad (4)$$

in which:

X = Magnitude of an event (logarithm)
x = X - M, deviation of a single event from the mean
N = Number of events in the record

b. The types of cumulative frequency curves fitted in hydrologic engineering do not require moments of a higher order than these three, and ordinary fitting will require only the first two. There is no need to tabulate the individual deviation for each logarithm, as the second parts of equations 3 and 4 can be used to compute the standard deviation and skew coefficient much more rapidly, directly from the original logarithms. In computing these quantities in this manner, however, it is essential that the intermediate quantities in the computation be accurate to at least four decimal places, which is not practicable without an automatic desk calculator or electronic computer.

c. Computation of the maximum likelihood frequency curve coordinates from the computed mean and standard deviation, using the adopted skew coefficient (usually zero for peak discharges and for rainfall frequencies) is accomplished as described below by use of the following equation:

$$\text{Log } Q = M + kS \qquad (5)$$

4-03. ANNUAL-EVENT CURVES

Frequency curves of annual maximum or minimum events are computed as follows (See exhibit 4 for example):

a. Mean Logarithm. After tabulation of the data in chronological order or in the order of magnitude, the logarithm (exhibit 41) of each discharge is tabulated to two decimal places. The mean logarithm is obtained by dividing the sum of these logarithms by the number of events (equation 2). Time can be saved by obtaining the sum of the logarithms on one register of a calculator at the same time that the sum of the squares of the logarithms are obtained on a second register for step (b).

b. Standard Deviation. The standard deviation is computed (equation 3) as follows:

(1) Obtain the sum of the squares of the logarithms in an automatic calculator. This quantity should not be rounded off, but all figures carried in the computation.

(2) The sum of the logarithms obtained in the same machine operation, which figure is also not to be rounded off, is squared and divided by the number of events. This is a single machine operation, and the quotient should be carried to as many places as is the sum of the squares.

(3) This quotient is subtracted from the sum of the squares to obtain a quantity numerically equal to the sum of the squares of the deviations from the mean.

(4) Divide this quantity by one less than the number of events. The square root of the quotient is the standard deviation.

When an automatic calculator is not available, steps (1), (2) and (3) can be replaced, as illustrated on exhibit 4, by the following:

(1.1) Tabulate to two decimal places the difference between each logarithm and the mean logarithm. This quantity is called the deviation.

(2.1) Tabulate the square of each deviation to three decimal places.

(3.1) Add the squares of the deviations.

c. Skew Coefficient. It is impractical to base the skew coefficient to be used in a frequency study on a single record of annual flows that is less than 100 years in length. Even if such a long record is available, it is possible that a more accurate determination of skew coefficient can be obtained by combining the information from other records. Unless the data show a radical departure from usual values of skew, zero skew coefficient should be used for frequency curves of annual maximum peak flows or precipitation, and coefficients given in paragraph 6-03 should be used for frequency curves of annual maximum flood volumes. Any radical departure of the observed data from the adopted skew coefficients would appear when the curve and data are plotted graphically. Special regional determination of skew coefficients are discussed in paragraph 7-11.

d. Computation of Curve. Each frequency curve can be computed as follows (See exhibit 7 for example):

(1) For selected values of P_{∞}, tabulate values of k obtained from exhibit 39 corresponding to the adopted skew coefficient.

(2) Multiple each of these by the computed standard deviation, and add each product in turn to the mean logarithm (Equation 5).

(3) Tabulate values of P_N from exhibit 40 corresponding to the selected P_∞ values

(4) Plot the antilogarithm of each of the sums obtained in step 2 against the selected P_N values obtained in step 3.

4-04. PARTIAL-DURATION CURVES

Once a frequency curve of annual events has been established, the corresponding partial-duration curve can be determined analytically by use of average criteria derived by Walter Langbein. These criteria are based on the assumption that there are a large number of flood events each year and that these events are mutually independent. They should not be used without checking their applicability unless only very approximate results are desired and time does not permit a more accurate determination. An average relationship developed empirically from many stations in a region would ordinarily be preferred, because experience indicates that the observed relationship is often different from the theoretical relationship (as demonstrated in reference 20). A summary of the Langbein criteria is contained in the following tabulation, and an example of its use is shown on exhibit 20.

Corresponding Exceedence Frequencies per Hundred Years

Annual-event curve (No. of years flow is exceeded per hundred years)	Partial-duration curve (No. of times flow is exceeded per hundred years)
1.00	1.00
2.00	2.02
5.0	5.1
10.0	10.5
20	22.3
30	35.6
40	51.0
50	69.3
60	91.7
63.2	100
70	120
80	161
90	230
95	300

4-05. USE OF HISTORICAL DATA

Where one or more large historical floods have been estimated, such items can be used as a guide in drawing the frequency curve, particularly in the range of higher flows. Such adjustment would ordinarily be done graphically, and the historical flows would be plotted in the order of magnitude in which they are known to have occurred, using the entire period of history (not only that period dating from the earliest known flood) for the computation of plotting positions. While it is important that all known information be used in the construction of a frequency curve, it should be recognized that historical estimates are not as valuable as comparable recorded flows, and that the largest known floods are not always representative for the period. If historical flows are particularly outstanding relative to recorded data, procedures illustrated on exhibits 11 and 12 can be used to compute a composite frequency curve. This consists of selecting plotting positions based on the flood magnitudes and periods of record and of historical knowledge, converting these plotting positions to linear distances as measured on probability grid (k values on exhibit 36), and solving for a best-fit slope by use of equation 36. This techique is explained fully in reference 23.

4-06. USE OF FLOW ESTIMATES

As discussed in paragraph 3-03b, use of a large number of flow estimates based on a record at a nearby location might lead to erroneous frequency estimates. However, there are some cases where one or a few estimates are essential. In the case where a record is not obtained because the gage was washed out, it is imperative that some estimate of the value be used. The fact that the estimate may be in error by 25 percent is minor compared to the error introduced by omitting the value from the record. Also, where a comprehensive flood volume-duration series is being studied (see section 6), and a few of the items are missing, it is ordinarily advantageous to estimate these items rather than to have a different number of items for each duration studied.

4-07. ABNORMAL DRY-YEAR EFFECTS

The shape of the frequency curve is sometimes seriously distorted by the dominance of minor runoff factors during dry years. This is particularly true where (a) floods are normally caused by rainstorms, yet high base flows from underground sources or occasional snowmelt prevent true measures of rainflood runoff during very dry years, or (b) floods are normally caused by rainstorms, but excessive diversion or channel losses deplete flows during dry years so that the frequency curve drops sharply at the lower end. In such cases, it may be advantageous to fit only the upper half of the annual floods with a theoretical frequency curve. A suggested procedure, illustrated on exhibits 13 and 14, simply converts plotting positions to a linear distance as measured on probability grid (k values on exhibit 36) and solves for a bestfit slope (standard deviation) by use of equation 36.

4-08. USE OF SYNTHETIC FLOODS

a. Frequency estimates at best are not fully reliable. Although theoretical devices are employed to extrapolate frequency curves beyond experienced values, such extrapolation is highly dangerous. It is sometimes possible to use synthetic floods for constructing a frequency curve or especially for extrapolating one more accurately.

b. Where runoff records are not available, an attempt is sometimes made to compute floods that would result from various rainfall amounts, and then to construct a synthetic flood frequency curve, using the rainfall frequency curve as a guide. This method is considered satisfactory for airport drainage design and for urban storm drain design, but is ordinarily not satisfactory where infiltration losses are a considerable percentage of precipitation. In these latter cases, it is best to construct synthetic frequency curves from regional correlation studies as discussed in section 7.

c. A large hypothetical flood can sometimes be used as a guide in extrapolating the frequency curve. Where it appears that major storms may have accidentally missed the basin considered, a major observed storm might be transposed to the basin, and the flood resulting from this storm on wet ground conditions could be assigned a reasonable frequency and used as an "anchor point" for extrapolating the frequency curve.

d. While the probable maximum flood is defined as the largest flood that is reasonably possible at a location, it would not be said that a larger flood could absolutely not occur. The science of meteorology and hydrology has not yet advanced to the stage where an absolute maximum estimate can be made. Consequently, it is not considered necessary that a frequency curve be limited to values smaller than the probable maximum flood. However, it is considered that such a flood would have an exceedence interval considerably in excess of 1,000 years.

4-09. SPECIAL GEOGRAPHIC CONSIDERATIONS

a. New England. Frequency studies of peak flows made in the New England area have indicated that the frequency curves have a strong positive skew. In studying this problem, two peculiar conditions were observed:

(1) The period of the last 40 years, which includes the period of record at most of the stream gaging stations, has an abnormally large number of major floods in comparison with the 100-year period covered by the longer records and in comparison with the 300-year period of history.

(2) Major floods occur from at least two independent causes, tropical hurricane storms and extratropical cyclones. Hurricane floods are comparatively rare, but produce extreme flows, and therefore cause an upward curvature of the frequency curve of annual maximum flows. Some improvement in frequency estimates in this region is attained by segregating hurricane and non-hurricane floods (see reference 21). However, this apparently does not solve entirely the problem of upward curvature of the frequency curves.

b. Desert Regions. In many desert areas, streamflow from general storms is ordinarily moderate each year, but an occasional intense thunderstorm may center over a basin and cause an exceptionally large flood. As in the case of the New England studies, floods from these two different causes tend to produce a sharp upward curvature of the frequency curve. While a satisfactory solution to this problem has not been attained, a reasonable approach appears to be to estimate the frequency of thunderstorms on a regional basis and if desired, to combine the estimated frequency of thunderstorm flows with the frequency of flows from general storms. In some basins, an entire year may pass with zero runoff. When analytical methods using flow logarithms are used, this presents a particular difficulty, since the logarithm of zero is minus infinity. It may be best to omit such years from the record, compute a tentative frequency curve based on the remaining years, and then adjust the exceedence frequency by the ratio of the number of years of record to the number of years with runoff. However, factors resulting in zero flow usually also affect small flows to the extent that the shape of the frequency curve is distorted in the range of lower flows. It is possible to compute only the upper half of the frequency curves, in such cases, using procedures described in paragraph 4-07.

c. Regions of Extreme Variance. In several regions in the United States, runoff frequency curves are very steep. This occurs particularly in the Texas escarpment area and in the Southern California-Arizona area. In extrapolating these frequency curves beyond the range of experienced floods by analytical means, unreasonably high flood estimates may result. In these cases, extra care must be used in extrapolation, and hypothetical computed floods might well be used as a general guide.

SECTION 5 - FLOOD PEAK FREQUENCY - ANALYTICAL ADJUSTMENT

5-01. INTRODUCTION

In most cases of frequency studies of runoff or precipitation, there are locations in the region where records have been obtained over a long period. Additional record at a nearby station is useful for extending the record at the site insofar as there is correlation between recorded values at the two locations.

5-02. ESTIMATING INDIVIDUAL EVENTS

It is possible by correlation or other means to estimate from the base station values, the individual events that were not recorded at the site. In doing this by use of regression methods discussed in section 9, however, the variance of the estimated values is reduced by the amount of non-determination between the two stations. In frequency studies, therefore, missing events should not be freely estimated by regression analysis (and probably not by graphical relations). In order to insure that such estimates do not unduly reduce the computed standard deviation, an estimate of an annual maximum flow Q_1 at station 1 based on a corresponding annual maximum flow Q_2 at a base station would be made for this purpose by the following equation:

$$X_1 - M_1 = (X_2 - M_2)S_1/S_2 \qquad (6)$$

in which X represents the logarithm of a discharge Q, S_1, and S_2 are the standard deviations of the logarithms of annual maximum flows for concurrent periods at stations 1 and 2 respectively and M_1 and M_2 are corresponding mean values of annual maximum logarithms.

5-03. DEGREE OF CORRELATION

The direct means of estimating the degree of correlation between corresponding flows at two stations is to arrange successive pairs of annual maximum flows in parallel columns for the concurrent period of record. These flows should not be arranged in the order of magnitude, but should be paired in their chronological sequence. The correlation coefficient $\bar{R}$ is computed as discussed in paragraph 9-03 by use of the following equations:

$$1 - \bar{R}^2 = (1 - R^2)\,\frac{N-1}{N-2} \qquad (7)$$

$$R^2 = \frac{(\Sigma xy)^2}{\Sigma x^2 \Sigma y^2} = \frac{(\Sigma XY - \Sigma X \ \ \Sigma Y/N)^2}{[\Sigma X^2 - (\Sigma X)^2/N][\Sigma Y^2 - (\Sigma Y)^2/N]} \qquad (8)$$

Symbols are defined in paragraph 9-02 and in section 11. As discussed in paragraph 9-03e, a correlation coefficient computed in the above manner may be unduly high or low, depending on chance variation in the data. When many such correlations have been computed within a region, it may be possible to modify these by judgment or regional correlation procedures in such a way as to result in a more reliable correlation estimate. It should be remembered, however, that this correlation coefficient usually has a minor influence on the ultimate frequency determination, and extensive studies designed to improve its reliability might ordinarily not be warranted.

5-04. ADJUSTMENT OF FREQUENCY STATISTICS

In cases where frequency curves are calculated analytically, it is neither necessary nor desirable to establish individual flows based on nearby stations, but adjustments can be made in the calculated statistics as follows:

$$S_1' - S_1 = (S_2' - S_2)\,R^2\,\frac{S_1}{S_2} \qquad \text{(approx.)} \qquad (9)$$

$$M_1' - M_1 = (M_2' - M_2)\,R\,\frac{S_1}{S_2} \qquad (10)$$

in which the primes indicate the long-period values, and those without primes are based on the same short period for both stations. Subscripts indicate the station number. An example of these adjustments is shown on exhibit 5.

5-05. ADVANTAGE OF ADJUSTMENT

The reliability of an adjusted value may be expressed in terms of the equivalent length of record required to establish an equally reliable unadjusted value. The equivalent record derived from a nearby station is obtained as follows:

$$N_1' = \frac{N_1}{1 - \frac{N_2' - N_1}{N_2'} \bar{R}^2} \quad \text{(approx.)} \qquad (11)$$

Thus, on exhibit 5, use of an additional 17 years of record at the base station is equivalent to adding about 9 years at the site, inasmuch as the coefficient of determination is 0.67.

5-06. SUMMARY OF PROCEDURE

The procedure for computing a frequency curve using data recorded at the site and at a nearby long-record station is summarized in the first five steps in paragraph 7-10 and the four steps in paragraph 4-03d.

SECTION 6 - FLOOD VOLUME FREQUENCY

6-01. NATURE AND PURPOSE

The comprehensive flood volume-duration frequency series consists of a set of frequency curves as follows:

a. Maximum rate of flow for each water year.
b. Maximum 1-day average flow for each water year.
c. Maximum 3-day average flow for each water year.
d. Maximum 10-day average flow for each water year.
e. Maximum 30-day average flow for each water year.
f. Maximum 90-day average flow for each water year.
g. Average flow for each water year.

Runoff volumes are expressed as average flows in order that peak flows and volumes can be readily compared and coordinated. Whenever it is necessary to consider flows separately for a portion of the water year such as the rain season or snowmelt season, the same items (up to the 30-day or 90-day values) are selected from flows during that season only. A comprehensive flood volume-duration series is used primarily for reservoir design and operation studies, and should be developed in the design of reservoirs having flood control as a major function. When reservoir problems involve runoff durations greater than one year, frequency studies might well include multi-annual runoff volumes and consideration of seasonal effects, as discussed in paragraph 6-06.

6-02. DATA FOR COMPREHENSIVE SERIES

Data to be used for a comprehensive flood volume-duration frequency study should be selected from complete water-year records in accordance with rules contained in paragraph 3-03. Unless overriding reasons exist, durations specified in paragraph 6-01 should be used in order to assure consistency among various studies for comparison purposes. Peak flows should be selected only for those years when recorder gages existed or when peak flows were measured by other means. Where a minor portion of a water-year's record is missing, the longer-duration flood volumes for that year can often be estimated adequately. Where upstream regulation or diversion exists, care should be exercised to assure that each period selected is that when flows would have been maximum under the specified (usually natural) conditions. The dates and amounts of each selected average flow should be tabulated in chronologic order in c.f.s. or thousand c.f.s.. A typical tabulation is illustrated on exhibit 15.

6-03. STATISTICS FOR COMPREHENSIVE SERIES

The analytical method used for flood volume-duration frequency computations is based on fitting the Pearson Type III function by use of moments of flow logarithms. In practice, only the first two moments, computed by use of equations 2 and 3, are based on station data. As discussed in paragraph 4-03, the skew coefficient should not be based on a single record, but should be derived from regional studies. The following coefficients, based on studies summarized in references 12 and 20, are considered to be generally applicable for annual maximum flood volume frequency computations:

Duration	Skew Coefficient
Instantaneous	0
1 day	-.04
3 days	-.12
10 days	-.23
30 days	-.32
90 days	-.37
1 year	-.40

A sample computation of frequency statistics is given on exhibits 15 and 19. This can be accomplished in steps as follows:

a. Tabulate annual-maximum average flows for each duration in chronological order as shown on exhibit 15.

b. Tabulate logarithms in chronological order as shown on exhibit 17. If a long-record station nearby is available, tabulate corresponding logarithms for that base station. The work thus far can be checked approximately by ascertaining that the logarithm for each succeeding duration decreases, but by not more than about 0.5. Much inconvenience can be eliminated by using data for the same years for each duration insofar as is feasible. If a year's record is incomplete, the missing portion can usually be estimated satisfactorily.

c. By use of a statistical calculator, the sums of each column of logarithms, the sums of their squares and the sum of their cross-products (doubled) can be obtained for each duration in a single cumulative operation, as described in paragraph 9-02b.

These should be entered on exhibit 18. In cases of peak flows, advantage can sometimes be gained by using 1-day values at the same station as a base instead of peak values at the base station, particularly if the records of peak flows are incomplete. The extended 1-day statistics would then be used as long-term values.

d. Compute the means, standard deviations and determination coefficients for each duration. If peak-flow statistics are extended by use of 1-day flows at the same station, possible inconsistencies can be avoided by using a determination coefficient of 1.00 instead of calculating the coefficient. Enter the long-term mean and standard deviation for each duration at the base station and compute the long-term mean and standard deviation for each duration for the station concerned. These operations are shown on exhibit 18.

e. Plot the extended standard deviation against the extended mean, and adopt a smooth relationship, as shown on exhibit 19. Tabulate these values on exhibit 18.

f. When many peak flows are missing from the record, it is best to add the average difference between corresponding peak and 1-day logarithms to the extended 1-day mean in order to obtain the peak mean logarithm. The peak standard deviation can then be obtained by extrapolation of the curve similar to that shown in exhibit 19.

g. Select a skew coefficient for each duration from the above tabulation or from special regional studies as discussed in paragraph 7-11. Tabulate on exhibit 18.

6-04. FREQUENCY CURVES FOR COMPREHENSIVE SERIES

a. General procedure. Frequency curves of flood volumes are computed analytically using general principles and methods of section 4. They should also be shown graphically and compared with the data on which they were based. This is a general check on the analytic work and will ordinarily reveal any inconsistency in data and methodology. Data are plotted on a single sheet for comparison purposes, using procedures described in section 3.

b. Computation of basic curves. Frequency curves are obtained from the frequency statistics and compared with observed frequencies for each of the seven basic durations as follows:

(1) Tabulate the average flows for each duration in order of magnitude and obtain the plotting position for each event from exhibit 37. This operation is shown on exhibit 16.

(2) Compute flows corresponding to related values of P_∞ for each frequency curve from the adjusted means and smoothed standard deviations, using equation 5 and coefficients obtained from exhibit 39, as shown on exhibit 18.

(3) Tabulate values of P_N from exhibit 40 for each selected value of P_∞ , using the average value of N' for all durations.

(4) Plot the points obtained in step (1) and the curves from coordinates obtained in steps (2) and (3) as shown on exhibit 20.

c. Interpolation between fixed durations. The runoff volume for any specified frequency can be determined for any duration between 24 hours and 1 year by drawing a curve on logarithmic paper as illustrated on exhibit 21, relating volume to duration for that specified frequency, using maximum 24-hour criteria derived in reference 15 and summarized in terms of average flows as follows:

$$\log Q_{24\text{-hr}} = 0.77 \log Q_{1\text{-day}} + 0.23 \log Q_{peak} \qquad (12)$$

When runoff volumes for durations shorter than 24 hours are very important, special frequency studies should be made. These could be done in the same manner as for the longer durations, using skew coefficients interpolated in some reasonable manner between those used for peak and 1-day flows. An approximate determination of short-duration frequencies is illustrated in reference 15.

6-05. DURATIONS EXCEEDING 1 YEAR

a. Introduction. In the design of reservoirs for conservation purposes (and occasionally for flood control purposes), the volumes of runoff that can be expected to occur during the lifetime of the structure within durations exceeding one year can be of primary concern. In general, the design of such reservoirs has been based on the lowest (or highest) volume of runoff observed for the critical duration during the period of record, which generally encompasses 40 to 100 years.

Frequently, however, it is felt that the observed minimum or maximum is much more extreme than should normally be anticipated in a similar future period. In order to determine expected volumes with a greater degree of reliability, frequency studies of long-duration volumes can be made. However, direct frequency analysis is ordinarily not practicable, because a relatively small number of independent long-duration volumes is contained in a single record. (Only 20 independent 5-year volumes are contained in a 100-year record, for example.) In order to overcome this limitation, a study has been made under the Civil Works Investigation program of the Corps of Engineers to relate long-duration volumes to annual volumes. Using criteria developed in this study, long-duration volumes can be derived from a frequency curve of annual volumes and the correlation coefficient between successive annual flows. Where this correlation coefficient is considered to be zero, the criteria should be fairly dependable, but where there appears to be substantial correlation between successive annual runoff values, the reliability decreases, principally because of the uncertainty as to the true correlation coefficient. A description of the study and the derived criteria is contained in reference 18.

b. Annual runoff. Criteria for determining multi-annual runoff are based on the logarithmic mean and standard deviation of annual runoff. These should be computed as described in paragraph 6-04.

c. Persistence effects. In many river basins, surface or subsurface storage effects cause the flows in one year to reflect conditions in the preceding year to some extent. This will result in a positive correlation between successive years' runoff. A measure of this persistence effect is the correlation coefficient, or preferably its square, the determination coefficient, between successive years' runoff logarithms. This is determined by pairing each year's runoff logarithm (except the first) with that of the preceding year, and computing the determination coefficient by use of equations 7 and 8. Because of the important effect of the determination coefficient on long-duration volume estimates, its degree of unreliability as discussed in paragraph 10-04 should be given special consideration.

d. Multi-annual runoff. A frequency curve of total runoff volume expected to occur during a period consisting of an integral number of water years can be obtained directly from the mean and standard deviation of annual (water-year) runoff logarithms and the determination coefficients between successive annual runoff logarithms, by use of chart 14 of reference 18. In using such a frequency curve, it should be remembered that there are (N - T + 1) T-year periods in an N-year record. For example, only 46 different 5-year volumes can occur in a 50-year period.

e. Seasonal effects. It must be recognized that multi-annual runoff based on water-year volumes is not representative of critical conditions. A drier or wetter period of the same duration can usually be found by starting the period some days or months earlier or later. Also, an integral number of years does not represent a critical duration, because adding one more dry season (or wet season) will ordinarily worsen the condition. For these reasons, criteria for determining maximum or minimum runoff volumes for any duration between 1 and 20 years and for any frequency were derived, as described in reference 18, and are summarized on chart 15 of that report.

f. Illustrative example. A sample computation of runoff volume frequencies for durations longer than 1 year is illustrated on chart 16 of reference 18.

6-06. APPLICATIONS OF FLOOD VOLUME-DURATION FREQUENCIES

a. Volume-duration curves. The use of flood volume-duration frequencies in solving reservoir planning, design, and operation problems usually involves the construction of volume-duration curves for specified frequencies. These are drawn first on logarithmic paper for interpolation purposes, as discussed in paragraph 6-04c and illustrated on exhibit 21, and are then replotted on arithmetic grid as shown on the same exhibit. A volume-duration curve for durations longer than 1 year is illustrated on chart 17 of reference 18. A straight line on this grid represents a constant rate of flow (so many acre-feet per day). The straight lines on exhibit 21 represent a uniform flow of 2,000 c.f.s., and placement on the 100-year volume-duration curve demonstrates that a reservoir capacity of 36,000 acre-feet is required to control the indicated runoff volumes to a project release of 2,000 c.f.s.. The curve also indicates that durations of 4 to 7 days are critical for this project release and flood control space.

b. Simple reservoir problems. In the case of a single flood control reservoir located immediately upstream of the only important damage center concerned, the volume frequency problems are relatively simple. A series of volume-duration curves similar to that shown on exhibit 21 corresponding to selected frequencies should first be drawn. The project release rate should be determined, giving due consideration to possible channel deterioration, encroachment into the flood plain, and operational contingencies. Lines representing this flow rate are then drawn tangent to each volume-duration curve, and the intercept in each case determines the reservoir space used to control the flood of that selected frequency. The point of tangency represents the critical duration of runoff. This procedure can be used not only as an approximate aid in selecting a reservoir capacity, but as an aid in drawing filling-frequency curves.

c. Complex reservoir problems. Where reservoir operation schedules are variable or where many reservoirs are operated jointly, it may be necessary to route historical flows month by month or day by day in order to demonstrate the adequacy of a design or operation procedure. However, the techniques described in the preceding paragraph may be applicable approximately, and may shed considerable light on the problem. In applying such techniques, the following guides should be used:

(1) Volume-duration curves are needed for unregulated flows at each important damage center.

(2) The straight line corresponding to the average nondamaging flow, allowing for operational contingencies, when drawn tangent to the volume-duration curve corresponding to the selected design frequency will indicate the storage required in the system if it is located and operated so as to be fully effective.

(3) The same straight line will indicate the range of critical durations for design and operation studies. A system of reservoirs should be "tuned" to this range of durations, insofar as is feasible, because a reservoir that fills and empties in 5 days may be of no value if the critical duration at a downstream damage center is 15 days. Likewise, a reservoir that is only half full in 15 days would not have provided its best control at the damage center.

d. Representative hydrographs. In solving complex reservoir problems, representative hydrographs at all locations can be patterned after one or more past floods. The ordinates of these hydrographs can be adjusted so that their volumes for the critical durations will equal corresponding magnitudes at each location for the selected frequency. A design or operation scheme based on regulation of such a set of hydrographs would be reasonably well balanced.

SECTION 7 - REGIONAL FREQUENCY ANALYSIS

7-01. GENERAL

Runoff frequency estimates for any specific location based on data recorded at that location and, where practicable, adjusted by use of longer-record stations nearby as described in section 5, can ordinarily be improved by a study of frequency characteristics throughout the hydrologic region. Such a regional study will also help to assure consistency of estimates for different locations and will provide means for estimating frequencies at locations where data are not available. Where time is limited and approximate estimates are satisfactory, some very simple schemes are helpful. Where time permits, more elaborate schemes are often justified.

7-02. USE OF FREQUENCY STATISTICS

A regional frequency correlation study is based on the two principal frequency statistics - the mean and standard deviation of annual maximum flow logarithms. Prior to relating these frequency statistics to drainage-basin characteristics, it is essential that the best possible estimate of each frequency statistic be made. This is done by adjusting short-record values by the use of longer records at nearby locations. When many stations are involved, it is best first to select long-record base stations for each portion of the region. It might be desirable to adjust the base station statistics by use of the one or two longest-record stations in the region, and then adjust the short-record station values by use of the nearest or most appropriate base station. Methods of adjusting statistics are discussed in section 5.

7-03. SIMPLE SCHEMES

For preliminary studies where a high degree of accuracy is not required, regional frequency analyses might consist simply of plotting the standard deviation against drainage area size or, for various locations on the same river, against river mile distance, if preferred. Similarly, the mean logarithm representing general magnitude can be plotted against drainage area size. Another simple scheme is to plot the standard deviation or mean logarithm of discharge per square mile on a map, and drawing lines of equal standard deviation or mean logarithm. Such an analysis can be used to modify the estimates slightly to improve consistency and to select statistics for ungaged areas in the region. This type of analysis is illustrated in reference 20.

7-04. DRAINAGE-BASIN CHARACTERISTICS

A regional analysis involves the determination of the main factors responsible for differences in precipitation or runoff regimes between different locations. This is done by correlating important factors with the long-record mean and with the long-record standard deviation of the frequency curve for each station. (The long-record values are those based on extension of the records as discussed in section 5.) Statistics based on rainfall measurements may be correlated in mountainous terrain with the following factors:

a. Elevation of station
b. General slope of surrounding terrain
c. Orientation of that slope
d. Elevation of windward barrier
e. Exposure of gage
f. Distance to leeward controlling ridge

Statistics based on runoff measurements may be correlated with the following factors:

a. Drainage area (contributing)
b. Slope of drainage area or of main channel
c. Surface storage (lakes and swamps)
d. Mean annual rainfall
e. Number of rainy days per year
f. Infiltration characteristics
g. Stream length

7-05. CORRELATION METHODS

Correlation methods and their application are discussed in section 9.

7-06. LINEAR RELATIONSHIPS

In order to obtain satisfactory results using multiple linear correlation techniques, all variables must be expressed so that the relation between the dependent and any independent variable can be expected to be linear, and so that the interaction between two independent variables is reasonable. An illustration of the first condition is the relation between rainfall and runoff. If the runoff coefficient is sensibly constant, as in the case of urban or airport drainage, then runoff can be expected to bear a linear relation to rainfall. However, in many cases initial losses and infiltration losses cause a marked curvature in the relationship. Ordinarily, it will be found that the logarithm of runoff is very nearly a linear function of rainfall, regardless of loss rates, and in such cases, linear correlation of logarithms would be most suitable. An illustration of the second condition is the relation between rainfall, D, drainage area, A, and runoff, Q. If the relation used for correlation is:

$$Q = aD + bA + c \tag{13}$$

then it can be seen that one inch change in precipitation would add the same amount of flow, regardless of the size of drainage area. This is not reasonable, but again a transformation to logarithms would yield a reasonable relation:

$$\text{Log } Q = d \log D + e \log A + \log f \tag{14}$$

or transformed:

$$Q = fD^dA^e \tag{15}$$

Thus, if logarithms of certain variables are used, doubling one independent quantity will multiply the dependent variable by a fixed ratio, regardless of what fixed value the other independent variables have. This particular relationship is reasonable and can be easily visualized after a little study. There is no simple rule for deciding when to use the logarithmic transformation. It is only feasible, however, when the variable has a fixed lower limit of zero.

7-07. EXAMPLE OF REGIONAL CORRELATION

An illustrative example of a regional correlation analysis of standard deviation with drainage area and number of rainy days per year is given on exhibit 22. Since many important variables are neglected, the analysis is not of the scope necessary for a complete study, but is useful for illustrating various techniques and problems involved in such a study. In the example, X_1 is one plus the logarithm of the adjusted standard deviation (one is added to eliminate negative values), X_2 is the logarithm of the drainage area size, and X_3 is the logarithm of the average number of rainy days per year for the drainage area. The regression equation is derived as shown, and the calculted coefficient of determination is 0.31, which means that 31 percent of the variance of X_1 is explained by the regression equation.

7-08. SELECTION OF USEFUL VARIABLES

In the regression equation derived on exhibit 22, the coefficient of X_2 is very small, which indicates that this factor has very little effect. To determine the usefulness of this factor, it is necessary to make an additional analysis using all variables except this one. In this case, the problem would resolve into a simple correlation analysis using X_3 of exhibit 22 as X_2 in equations 19 to 21. Then:

$$b_2 = -1.1749/2.3484 = -0.50 \tag{16}$$

and

$$a = 0.3578 - (-.50)1.925 = 1.32 \tag{17}$$

Hence

$$X_1 = 1.32 - 0.50X_3 \tag{18}$$

A solution for $\overline{R}^2$ (equations 31 and 33) would yield 0.32. Thus, a better correlation is obtained neglecting drainage area as a factor. If additional factors were considered in the analysis, the effect of drainage area should be reconsidered, as it is possible that its effect is obscured in the example by neglecting some other important variable. The final test of importance of a particular factor is a comparison of the correlation coefficient using all factors and then omitting only the factor whose influence is being tested. Even in the case of a slight increase in correlation obtained by adding a variable, consideration of the increased unreliability of $\overline{R}$, as discussed in paragraph 10-04, might indicate that such factor should be eliminated in cases of small samples.

7-09. USE OF MAPS

Many hydrologic factors cannot be expressed numerically. Examples are soil characteristics, vegetal cover, and geology. For this reason, numerical regional analysis will explain only a portion of the regional variation of runoff frequencies. The remaining unexplained variance is contained in the regression constant, which can be considered to vary from station to station. These regression constants (residuals) can be computed by inserting the drainage basin characteristics and frequency statistic for each station in the regression equation and solving for the regression constant. These constants can be plotted on a regional map, and lines of equal values drawn (perhaps using soils or vegetation maps as a guide). Use of such a map for selecting a regression constant should be much better than using the single constant for the entire region derived from equation 28. In smoothing lines on such a map, consideration should be given to the reliability of computed statistics. Exhibit 23 shows standard errors of estimating means and standard deviations. As an example, if a computed standard deviation based on 30 years of record is 0.300, there is about one chance in 20 that the mean is in error by more than 0.110 (twice the standard error) or that the standard deviation is in error by a factor of 1.3 (antilog of 0.114).

7-10. SUMMARY OF PROCEDURE

A regional analysis of precipitation or floodflows is accomplished in the following steps:

a. Select long-record base stations within the region as required for extension of records at each of the short-record stations.

b. Tabulate maximum events of each station, corresponding logarithms, and logarithms of base-station values for the corresponding years. Logarithms should be rounded to 2 decimal places.

c. Calculate M and S (equations 2 and 3) for each base station.

d. Calculate M and S for each other station and for the corresponding values of the base station, and calculate the correlation coefficient (equations 31 and 33). Summation of logarithms and their squares for both stations and their cross-products can be obtained in a single cumulative operation of an automatic calculator, as discussed in paragraph 9-02B.

e. Adjust all values of M and S by use of the base station, (equations 9 and 10). (If any base station is first adjusted by use of a longer-record base station, the longer-record statistics should be used for all subsequent adjustments.)

f. Select meterological and drainage basin parameters that are expected to correlate linearly with M and log S, and tabulate estimated values of these for each area. (The physical significance of log S is not important, as the transformation simply eliminates a lower limit of zero from the regression variables.)

g. Calculate the regression equations relating M and log S in turn to these statistics, using procedures explained in section 9, and compute the corresponding determination coefficients.

h. Eliminate variables in turn that contribute the least to the determination coefficient, recomputing the determination coefficient each time, and select the regression equation having the highest determination coefficient, or one with fewer variables if the determination coefficient is about as high.

i. Compute the regression constants (residuals) for each station, plot on a suitable map, and draw isopleths of the regression constant for the regression equations of M and S (two maps), considering that the regression constant for a station represents a basin-mean value.

j. A frequency curve can be computed from constants obtained for any basin on the map, using the computed regression equations to obtain M and S, and using procedures discussed in paragraph 4-03 for computing a frequency curve therefrom.

7-11. REGIONAL SKEW DETERMINATIONS

Skew coefficients for use in hydrologic studies should be based on regional studies, since values based on individual records in the order of 100 years or less of length are highly unreliable. This can be done by computing skew coefficients for available records and using the average, weighted in accordance with record length. Unless an electronic computer is available, such procedure is laborious. An alternative procedure suitable for desk calculation is to compute an average skew coefficient as illustrated on chart 4 of reference 14.

7-12. SAMPLE REGIONAL CRITERIA

The regional frequency analysis described in reference 20 is considered to be a moderately elaborate type of analysis. The criteria derived are suitable for selecting frequency statistics for deriving frequency curves of runoff peaks and volumes for durations up to 30 days. Standard deviations for frequency curves for the various durations of runoff are obtained directly from maps constructed for each duration. The mean logarithms of runoff for each duration, however, have been related to drainage area size, normal annual precipitation, elevation, and by means by maps, to geographical location. To illustrate the relative simplicity of this scheme, criteria for determining frequency curves of peak flows are included herein as exhibits 24 to 27. Data and computations required for synthetic frequency curves for the drainage basin used in illustrating graphical and analytical methods on exhibits 2 to 4 are as follows:

Drainage basin characteristics (location shown on exhibit 24):

Drainage area	134 sq. mi.
Normal annual precipitation	47 in.
Average elevation	2900 ft. m.s.l.
Average latitude	40° - 03'

Frequency constants:

Standard deviation (from exhibit 25)	.31
C_p (from exhibit 24)	42
K (from exhibit 26)	.71

Computation of Q_p

$$Q_p = .001\ C_p A^{.85}\ P^2 K \quad \text{(equation 11, reference 20)}$$

$$= .001\ (42)\ (64)\ (47)^2\ (.71)$$
$$= 4220 \text{ c.f.s.}$$

Flow of specified frequency (say once per 100 years

$Q_{.01}$ = 5.7 (4,220) = 24,000 c.f.s. (from exhibit 27)

SECTION 8 - FREQUENCIES OF OTHER HYDROLOGIC FACTORS

8-01. INTRODUCTION

The frequency methods described in sections 3 and 4 have application to runoff, and can also be used in estimating frequencies of various other hydrologic factors. Some of the more common applications are described in the following paragraphs.

8-02. RAINFALL FREQUENCIES

Procedures for the computation of frequency curves of station precipitation, both graphically and analytically, are generally identical to those for streamflow analysis. In precipitation studies, however, instantaneous peak intensities are ordinarily not analyzed, since they are virtually impossible to measure and of little application. Precipitation amounts for specified durations are commonly analyzed, mostly for durations of less than three or four days. The few studies made thus far have indicated that the logarithmic normal function (with zero skew coefficient) is fitted fairly well with annual maximum station precipitation data, regardless of the duration used. Station precipitation alone is not adequate for most hydrologic studies, and some means of evaluating the frequency of simultaneous or near simultaneous precipitation over the area is necessary.

8-03. LOW FLOW FREQUENCIES

a. The design of hydroelectric powerplants and the design of reservoirs for supplementing low river flows for water quality and other purposes requires the evaluation of the frequencies of low flows for various durations. The method of frequency analysis previously discussed is usually applicable, except that minimum instead of maximum runoff for each period is selected from the basic data. In studying low flows, it will be found that the effects of basin development are relatively great. For example, a relatively moderate diversion can be neglected when studying floodflows, but might greatly modify or even eliminate low flows. Accordingly, one of the most important aspects of low flows concerns the evaluation of past and future effects of basin developments.

b. Civil Works Investigations Project No. 154 has been established by the Corps of Engineers to study low flows and their frequency, and has met with considerable success in applying analytical procedures described herein. These studies have not progressed sufficiently to provide firm criteria, however.

c. In regions of water scarcity and where a high degree of development has been attained, basin development projects that entail carryover of water for several years are often planned. A study of frequencies of flows having durations exceeding one year is contained in reference 18 and may be of considerable use in connection with such projects.

8-04. HURRICANE FREQUENCIES

In studies of hurricane wind velocities by the Corps of Engineers and U.S. Weather Bureau, the central pressure index (estimated minimum sea level pressure for individual hurricane) has been used in conjunction with pressure vs. wind relationships to determine wind frequencies. The index frequency for each of three large geographic zones was determined as illustrated on exhibit 28, and subsequently divided into frequencies for various subdivisions of each zone. The minimum hurricane pressure apparently plots close to a straight line on arithmetic probability paper.

8-05. SEDIMENT FREQUENCIES

Another illustration of use of frequency techniques discussed herein is shown on exhibit 29, where the frequency of annual sediment load of the Colorado River is shown to approximate a linear relationship on log probability paper.

8-06. COINCIDENT FREQUENCIES

a. In many cases of hydrologic design, it is necessary to consider only those events which occur coincidentally with other events. For example, a pumping station is usually required to pump water only when interior drainage occurs at a time that the main river stage is above the gravity outlet. In constructing a frequency curve of interior drainage flows that occur only at such times, data selected for direct use should be limited to that recorded during high river stages. In some cases, such data might be adequate, but it is usually possible in cases where the two types of events do not correlate to make indirect use of noncoincident data in order to establish a more reliable frequency curve of coincident events. The general procedure used is as follows:

(1) Select the more stable of the two variables whose coincidental frequency is to be determined. This will be designated as variable B, and the other as variable A.

(2) Determine the time limits of the seasons during which high stages of variable B are about uniformly likely. If all important stages of variable B can be limited to one season, the analysis will be simplified. Otherwise the following steps must be duplicated for each season.

(3) Compute a frequency curve of stages (or flows, rainfall amounts, etc.) for variable A, using data obtained only during the selected season.

(4) Construct a duration curve of stages (or flows, etc.) for variable B, using data obtained only during the selected season.

(5) The exceedence frequency of any selected magnitude of variable A that is coincidental with any specified range of stage for variable B is equal to the product of the exceedence frequency indicated by the curve derived in (3) and the proportion of time flows at B are within the selected range of stage, as indicated by the curve derived in (4).

b. Exhibit 30 illustrates a computation of reservoir stage or storage frequency curve from consideration of coincidental frequencies. In this hypothetical case of a 300,000 acre-foot reservoir, the top 100,000 acre-feet is reserved for flood control, the next 50,000 for seasonal irrigation requirements, and the remainder for carryover irrigation storage and power head. Because flood control space would rarely be used, routings of recorded floods do not adequately define the frequency of storage in the flood control space. The example shows a flood frequency curve in terms of the project design (50-year) flood, and a storage duration curve determined from monthly routings of recorded runoff. This latter curve represents only those months when major storms are likely to occur, and does not reflect reservoir rises that would occur during floods. Therefore, it represents conditions that can exist at the beginning of a flood. The duration curve was divided into four ranges, and average storages determined for each range. Routings (not shown) of various percentages of the project design flood with these four initial storages were made, and four stage-frequency curves drawn as shown. A composite frequency curve was then drawn as illustrated in the inset table.

SECTION 9 - CORRELATION ANALYSIS

9-01. NATURE AND APPLICATION

a. Correlation is the process of determining the manner in which the changes in one or more independent variables affect another (dependent) variable. The dependent variable is the value sought and is to be related to various independent variables which will be known in advance, and which will be physically related to the dependent variable. For example, the volume of spring runoff on a river (dependent variable) might be correlated with the depth of snow cover in the area (independent variable). Recorded values of such variables over a period of years might be plotted as a graph and the apparent relation sketched in by eye. However, correlation methods will generally permit a more dependable determination of the relation and have the additional advantage of providing means for evaluating the dependability of the relation or of estimates based on the relation.

b. The function relating the variables is termed the "regression equation," and the proportion of the "variance" of the dependent variable that is explained by the regression equation is termed the "coefficient of determination," which is the square of the "correlation coefficient." Regression equations can be linear or curvilinear, but linear regression suffices for most applications, and curvilinear regression is therefore not discussed herein.

9-02. CALCULATION OF REGRESSION EQUATIONS

a. In a simple correlation (one in which there is only one independent variable), the linear regression equation is written

$$X_1 = a + b_2X_2 \qquad (19)$$

in which X_1 is the dependent variable, X_2 is the independent variable, a is the regression constant, and b_2 is the regression coefficient. The coefficient b_2 is evaluated from the tabulated data by use of the equation

$$b_2 = \Sigma(x_1x_2)/\Sigma(x_2)^2 \qquad (20)$$

in which x_1 is the deviation of a single value X_1 from the mean M_1 of its series, and x_2 is similarly defined. The regression constant is obtained from the tabulated data by use of the equation

$$a = M_1 - b_2M_2 \tag{21}$$

b. All summations required for a simple linear correlation can be obtained in a single cumulative operation of a single-keyboard 10-bank automatic calculator, using equations 29 and 30, as follows:

(1) Round all values of X_1 to the same decimal place so that the median value has two significant figures. Logarithms should be rounded to two decimal places. Repeat for values of X_2. If there are negative values of any variable, add a constant to all values of that variable and subsequently subtract that constant from the mean.

(2) Enter the first value of X_1 on the left of the keyboard using banks 2, 3 and if necessary, bank 1. Enter the corresponding value of X_2 on the right of the keyboard using banks 9, 10 and if nceessary, bank 8. Square the quantity on the keyboard.

(3) Lock both the multiplier and product dials and repeat the process with each pair of values. When all values have been squared, the sum of the X_1 and X_2 values will appear on the multiplier dial, the sum of their squares will appear at each end of the product dial, and twice the sum of their cross-products will appear in the middle seven digits of the product dial. Care should be exercised that no cumulative product exceeds the machine capacity (7 digits, usually).

c. In a multiple correlation (one in which there is more than one independent variable) the linear regression equation is written

$$X_1 = a + b_2X_2 + b_3X_3 \ . \ . \ . + b_nX_n \tag{22}$$

In the case of two independent variables, the b coefficients are evaluated from the tabulated data by solution of the following simultaneous equations:

$$\Sigma(x_2)^2b_2 + \Sigma(x_2x_3)b_3 = \Sigma(x_1x_2) \tag{23}$$

$$\Sigma(x_2x_3)b_2 + \Sigma(x_3)^2b_3 = \Sigma(x_1x_3) \tag{24}$$

In the case of 3 independent variables, the b coefficients can be evaluated from the tabulated data by solution of the following simultaneous equations:

$$\Sigma(x_2)^2 b_2 + \Sigma(x_2 x_3) b_3 + \Sigma(x_2 x_4) b_4 = \Sigma(x_1 x_2) \quad (25)$$

$$\Sigma(x_2 x_3) b_2 + \Sigma(x_3)^2 b_3 + \Sigma(x_3 x_4) b_4 = \Sigma(x_1 x_3) \quad (26)$$

$$\Sigma(x_2 x_4) b_2 + \Sigma(x_3 x_4) b_3 + \Sigma(x_4)^2 b_4 = \Sigma(x_1 x_4) \quad (27)$$

For cases of more than three independent variables, the appropriate set of simultaneous equations can be easily constructed after studying the patterns of the above two sets of equations. In such cases, solution of the equations becomes tedious, and considerable time can be saved by use of the Crout method outlined in reference 2. Also, programs are available for solution of simple or multiple linear regression problems on practically any type of electronic computer.

<u>d.</u> For multiple regression equations, the regression constant should be determined as follows:

$$a = M_1 - b_2 M_2 - b_3 M_3 \ . \ . \ . \ -b_n M_n \quad (28)$$

<u>e.</u> In equation 20 and equations 23 - 27, the quantities $\Sigma(x)^2$ and $\Sigma(x_1 x_2)$ are obtainable rapidly by use of the equations

$$\Sigma(x)^2 = \Sigma(X)^2 - (\Sigma X)^2/N \quad (29)$$

$$\Sigma(x_1 x_2) = \Sigma(X_1 X_2) - \Sigma X_1 \ \ \Sigma X_2/N \quad (30)$$

9-03. THE CORRELATION COEFFICIENT AND STANDARD ERROR

<u>a.</u> The correlation coefficient is the square root of the coefficient of determination, which is the proportion of the variance of the dependent variable that is explained by the regression equation. A correlation coefficient of 1.00 would correspond to a coefficient of determination of 1.00, which is the highest theoretically possible and indicates that whenever the values of the

independent variables are known exactly, the corresponding value of the dependent variable can be calculated exactly. A correlation coefficient of 0.5 would correspond to a coefficient of determination of 0.25, which would indicate that 25 percent of the variance is accounted for and 75 percent unaccounted for. The remaining variance (error variance) would be 75 percent of the original variance and the remaining standard error would be the square root of this or 87 percent of the original standard deviation of the dependent variable. Thus, with a correlation coefficient of 0.5, the average error of estimate would be 87 percent of the average errors of estimate based simply on the mean observed value of the dependent variable without a regression analysis.

b. The correlation coefficient ($\bar{R}$) is determined by use of the following equations:

$$\bar{R}^2 = 1 - (1 - R^2)(N - 1)/df. \tag{31}$$

$$R^2 = \frac{b_2\ \Sigma(x_1x_2) + b_3\ \Sigma(x_1x_3)\ .\ .\ .\ + b_n\ \Sigma(x_1x_n)}{\Sigma(x_1)^2} \tag{32}$$

In the case of simple correlation, equation 32 resolves to

$$R^2 = \frac{(\Sigma x_1x_2)^2}{\Sigma x_1{}^2 \Sigma x_2{}^2} \tag{33}$$

c. The number of degrees of freedom (df), is obtained by subtracting the number of variables (dependent and independent) from the number of events tabulated for each variable.

d. The standard error (S_e) of a set of estimates is the root-mean-square error of those estimates. On the average, about one out of three estimates will have errors greater than the standard error and about one out of 20 will have errors greater than twice the standard error. The error variance is the square of the standard error. The standard error or error variance of estimates based on a regression equation is calculated from the data used to derive the equation by use of either of the following equations:

$$S_e^2 = \frac{\Sigma(x_1)^2 - b_2\Sigma(x_1x_2) - b_3\Sigma(x_1x_3) \; . \; . \; . \; . \; - b_n\Sigma\,(x_1x_n)}{df} \quad (34)$$

$$S_e^2 = (1 - \bar{R}^2)\Sigma(x_1)^2/(N - 1) \quad (35)$$

Inasmuch as there is some degree of error involved in estimating the regression coefficients, the actual standard error of an estimate based on one or more extreme values of the independent variables is somewhat larger than is indicated by the above equations, but this fact is usually neglected.

e. In addition to considering the amount of variance that is indicated by the correlation coefficient and standard error to be solved by the regression equation, it is important to consider the reliability of these indications. There is some chance that any correlation is accidental, but the higher the correlation and the larger the sample upon which it is based, the less is the chance that it would occur by accident. Also, the reliability of a regression equation decreases rapidly as the number of independent variables increases, and extreme care must be exercised in the use of multiple correlation in cases based on small samples.

9-04. SIMPLE LINEAR CORRELATION EXAMPLE

a. An example of a simple linear correlation analysis is illustrated on exhibits 31 and 32. The study from which this example was taken involved the determination of the areal distribution of short-duration precipitation in a mountainous region. Inasmuch as short-duration measurements were available at a relatively small number of locations, it was decided to investigate the relationship of short-duration to long-duration precipitation measurements, which were available at many locations.

b. Inasmuch as long-duration precipitation is made up of the sum of short-duration precipitation amounts, there is no question as to the existence of a physical relationship, and it is therefore obvious that the first requirement of a correlation analysis (logical physical relationship) is satisfied. Values of maximum recorded 12-hour precipitation and of mean annual precipitation were tabulated as shown on exhibit 31 and plotted as shown on exhibit 32. It was determined that the relation on logarithmic paper would logically approximate a straight line. Accordingly, the logarithms of the values were tabulated, and a linear correlation study of them made, as illustrated on exhibit 31, using equations given in paragraph 9-02. As the item to be calculated is the short-duration precipitation, the logarithm of that item is selected as the dependent variable (X_1).

c. The regression equation is plotted as curve A on exhibit 32. This curve represents the best estimate of what the maximum 12-hour precipitation would be at a location where the mean annual precipitation is known and the maximum 12-hour precipitation is not.

d. In addition to the curve of best fit, approximate reliability-limit curves are established at a distance of 2 standard errors from curve A. As logarithms are used in the regression analysis, the effect of adding (or subtracting) twice the standard error to the estimate is equivalent to multiplying (or dividing) the precipitation values by the antilogarithm of twice the standard error. In this case, the standard error is 0.081, and the antilogarithm of twice this quantity is 1.45. Hence, values of 12-hour precipitation represented by the limit curves are those of curve A multiplied and divided respectively by 1.45. In about 95 percent of all cases, the true value of the dependent variable will lie between these limit curves.

9-05. FACTORS RESPONSIBLE FOR NON-DETERMINATION

a. Factors responsible for correlations being less than 1 (perfect correlation) consist of pertinent factors not considered in the analysis and of errors in the measurement of those factors considered. If the effect of measurement errors is appreciable, it is possible in some cases to evaluate the standard error of measurement of each variable (see paragraph 9-03d) and to adjust the correlation results for such effects.

b. If an appreciable portion of the variance of X_1 (dependent variable) is attributable to measurement errors, then the regression equation would be more reliable than is indicated by the standard error of estimate computed from equations 34 or 35. This is because the departure of some of the points from the regression line on exhibit 32 is artificially increased by measurement errors and therefore exaggerates the unreliability of the regression function. In such a case, the curve is generally closer to the true values than to the erroneous observed values. Where there is large measurement error of the dependent variable, the error of regression estimates should be obtained by subtracting the measurement error variance from the error variance obtained from equation 34 or 35. If well over half of the variance of the points from the best-fit line is attributable to measurement error in the dependent variable, then the regression line would actually yield a better estimate of a value than the original measurement.

c. If appreciable errors exist in the values of the independent variable, the regression coefficient and constant will be affected, and fallacious estimates will result. Hence, it is important that values of the independent variable be rather accurately determined.

d. In the example used in paragraph 9-04, there may well be factors responsible for high short-time intensities that do not contribute appreciably to annual precipitation. Consequently, some locations with extremely high mean annual precipitation may have maximum short-time intensities that are not correspondingly high, and vice versa. Therefore, the station having the highest mean annual precipitation would not automatically have the highest short-time intensity, but would in general have something less than this. On the other hand, if mean annual precipitation were made the dependent variable, the station having the highest short-time intensity would be expected to have something less than the highest value of mean annual precipitation. Thus, by interchanging the variables, a change in the regression line is effected. Curve B of exhibit 32 is the regression curve obtained by interchanging the variables X_1 and X_2. As there is a considerable difference in the two regression curves, it is important to use the variable whose value is to be calculated from the regression equation as the dependent variable in those cases where some important factors have not been considered in the analysis. If it is obvious that all of the pertinent variables are included in the analysis, then the variance of the points about the regression line is due entirely to measurement errors, and the resulting difference in slope of the regression lines is entirely artificial. In cases where all pertinent variables are considered and the great preponderance of the measurement error is in one variable, that variable should be used as the dependent variable, as its errors will then not affect the slope of the regression line. In other cases where all pertinent variables are considered, an average slope should be used. An average slope can be obtained by use of the following equation:

$$b = \left[\frac{\Sigma x_1^{\,2}}{\Sigma x_2^{\,2}} \right]^{1/2} = \left[\frac{\Sigma X_1^{\,2} - (\Sigma X_1)^2 \,/\, N}{\Sigma X_2^{\,2} - (\Sigma X_2^{\,2} \,/\, N} \right]^{1/2} \qquad (36)$$

9-06. MULTIPLE LINEAR CORRELATION EXAMPLE

a. An example of a multiple linear correlation is illustrated on exhibit 33. In this case, the volume of spring runoff is correlated with the water equivalent of the snow cover measured on April 1, the winter low-water flow (index of ground water) and the precipitation falling on the area during April. Here again, it was determined that logarithms of the values would be used in the regression equation. Although the loss of 4 degrees of freedom of 12 available, as in this case, is not ordinarily desirable, the correlation attained (0.96) is particularly high, and the equation is consequently fairly reliable. Note that the column arrangement of the cross-product sums identical to their arrangement in the simultaneous equations.

b. In determining whether logarithms should be used for the dependent variable as above, questions such as the following should be considered: "Would an increase in snow cover contribute a greater increment to runoff under conditions of high ground water (wet ground conditions) than under conditions of low ground water?" If the answer is yes, then a logarithmic dependent variable (by which the effects are multiplied together) would be superior to an arithmetic dependent variable (by which the effects are added together). Logarithms should be used for the independent variables when they would increase the linearity of the relationship. Whenever logarithms are used, the logarithms should be taken of values that have a natural lower limit of zero and a natural upper limit that is large compared to the values used in the study.

c. It will be recognized that multiple correlation performs a function that is difficult to perform graphically. Reliability of the results, however, is highly dependent on the availability of a large sampling of all important factors that influence the dependent variable. In this case, the standard error of an estimate as shown on exhibit 33 is approximately 0.037, which, when added to a logarithm of a value, is equivalent to multiplying that value by 1.09. Thus, the standard error is about 9 percent, and the 1-in-20 error is roughly 18 percent. As discussed in paragraphs 9-03e and 10-04, however, the calculated correlation coefficient may be accidentally high. It can be demonstrated that the calculated correlation of the parameters as low as 0.89 (one chance in 20). With a correlation coefficient of 0.89, and therefore a coefficient of determination ($\bar{R}^2$) of 0.79, the standard error (from equation 35) would be 0.061. The antilogarithm is 1.15, so the true standard error might easily be almost double that estimated.

9-07. PARTIAL CORRELATION

The value gained by using any single variable (such as April precipitation) in a regression equation can be measured by making a second correlation study using all of the variables of the regression equation except that one. The loss in correlation is expressed in terms of the partial correlation coefficient, which is a measure of the decrease in error attributable to adding one variable to the correlation. The square of the partial correlation coefficient is obtained as follows:

$$r^2_{14.23} = \frac{(1 - \bar{R}^2_{1.23}) - (1 - \bar{R}^2_{1.234})}{1 - \bar{R}^2_{1.23}} \qquad (37)$$

in which the first subscript ahead of the decimal indicates the dependent variable and the second indicates the variable whose partial correlation coefficient is being computed, and the subscripts after the decimal indicate the independent variables. This procedure is fairly laborious except where electronic computers are used, and approximation of the partial correlation can be made by use of beta coefficients, which are very easy to obtain by use of the following equation after the regression equation has been calculated:

$$\beta_4 = b_4 \frac{S_4}{S_1} = b_4 \left[\frac{\Sigma x_4^2}{\Sigma x_1^2} \right]^{1/2} \qquad (38)$$

The beta coefficients of the variables are proportional to the influence of each variable on the result. While the partial correlation coefficient measures the increase in correlation that is obtained by additon of one more independent variables to the correlation study, the beta coefficient is a measure of the proportional influence of a given independent variable on the dependent variable. These two coefficients are related closely only when there is no interdependence among the various "independent" variables. However, some "independent" variables naturally correlate with each other, and when one is removed from the equation, the other will take over some of its weight in the equation. For this reason, it must be kept in mind that beta coefficients indicate partial correlation only approximately.

9-08. VERIFICATION OF CORRELATION RESULTS

Acquisition of basic data after a correlation study has been completed will provide an opportunity for making a check of the correlation results. This is done simply by comparing the values of the dependent variable observed, with corresponding values calculated from the regression equation. The differences are the errors of estimate, and their root-mean-square is an estimate of the standard error of the regression-equation estimates (paragraph 9-03). This standard error can be compared to that already established in equation 34 or 35. If the difference is not significant, there is no reason to suspect the regression equation of being invalid, but if the difference is large, the regression equation and standard error should be recalculated using the additional data acquired.

9-09. PRACTICAL GUIDE LINES

The most important thing to remember in making correlation studies is that accidental correlations occur frequently, particularly when the number of observations is small. For this reason, variables should be correlated only when there is reason to believe that there is a physical relationship. It is helpful to make preliminary examination of relationships between two or more variables by graphical plotting. This is particularly helpful for determining whether a relationship is linear and in selecting a transformation for converting curvilinear relationships to linear relationships. It should also be remembered that the chance of accidentally high correlation increases with the number of correlations tried. If a variable being studied is tested against a dozen other variables at random, there is a good chance that one of these will produce a good correlation, even though there may be no physical relation between the two. In general, the results of correlation analyses should be examined to assure that the derived relationship is reasonable. For example, if streamflow is correlated with precipitation and drainage area size, and the regression equation relates streamflow to some power of the drainage area greater than one, a maximum exponent value of one should be used, because the flow per square mile cannot increase with drainage area when other factors remain constant.

SECTION 10 - STATISTICAL RELIABILITY CRITERIA

10-01. FUNCTION

One of the principal advantages of the use of statistical procedures is that they provide means for evaluating the reliability of the estimates. This permits a broader understanding of the subject and provides criteria for decision making. The common statistical index of reliability is the standard error of estimate, which is defined as the root-mean-square error. In general, it is considered that the standard error is exceeded on the positive side one time out of six estimates, and equally frequently on the negative side, for a total of one time in three estimates. An error twice as large as the standard error of estimate is considered to be exceeded one time in 40 in either direction, for a total of one time in 20. These are only approximate frequencies of errors, and exact statements as to error probability must be based on examination of the frequency curve of errors or the distribution of the errors. Both the standard error of estimate and the distribution of errors will be discussed in this section.

10-02. RELIABILITY OF FREQUENCY STATISTICS

a. The standard errors of estimate of the mean, standard deviation, and skew coefficient, which are the principal statistics used in frequency analysis, are given in the following equations:

$$S_M = \sqrt{S^2/N} \tag{39}$$

$$S_s = \sqrt{S^2/2N} \tag{40}$$

$$S_g = \sqrt{6N(N-1)/(N-2)(N+1)(N+3)} \tag{41}$$

These have been used to considerable advantage as discussed in paragraph 7-09 in drawing maps of mean and standard deviation for a regional frequency study.

b. The distribution of errors of estimating the mean is a function of the t distribution, exhibit 34, and is given by the following equation:

$$\text{Prob}\left[\frac{|M-\mu|}{S}(N)^{1/2}>C\right] = \text{Prob}\left[t_{N-1}>C\right] \quad (42)$$

In any specific problem, values of N, M, and S are obtained from the recorded data. To find a value of $M-\mu$ (difference between computed and true mean) that is exceeded with a specified probability, ascertain from exhibit 34 the value of t corresponding to that probability, equate this to C and solve for $M-\mu$ by replacing the inequality sign by an equal sign in the left bracket of equation 42. The t distribution is symmetrical and therefore the probability that $M-\mu$ exceeds a specified value is equal to the probability that $\mu-M$ exceeds that same value, and therefore the probability that the absolute value of $M-\mu$ is exceeded regardless of direction is twice as great as indicated in exhibit 34. (Most published tables of t show twice the probabilities indicated in exhibit 34 and therefore represent both tails of the t distribution.)

c. The distribution of errors in estimating the standard deviation is a function of the chi-square distribution, exhibit 35, and is given by the following equation:

$$\text{Prob}\left[\frac{(N-1)S^2}{\sigma^2}>C\right] = \text{Prob}\left[\chi^2_{N-1}>C\right] \quad (43)$$

Here again N and S are obtainable from recorded data. To find a value of σ (true standard deviation) that is exceeded with a specified probability, ascertain from exhibit 35 the value of χ^2 (chi-square) that is exceeded with that probability, and compute σ by replacing the inequality sign in the left bracket of equation 43 by an equal sign. (Note that equation 43 is also valid if both inequality signs are reversed simultaneously.)

10-03. RELIABILITY OF FREQUENCY ESTIMATES

The reliability of analytical frequency determinations can best be illustrated by establishing error-limit curves. The error of the estimated flow for any given frequency is a function of the errors in estimating the mean and standard deviation, assuming that the skew coefficient is known. Criteria for construction of error-limit curves are based on the distribution of the "noncentral t". Selected values transformed for convenient use are given on exhibit 6. By use of this exhibit and equation 5, error-limit curves shown on exhibit 7 were calculated as illustrated on that exhibit. While the expected frequency is that shown by the middle curve, there is one chance in 20 that the true value for any given frequency is greater than that indicated by the .05 curve and one chance in 20 that it is smaller than the value indicated by the .95 curve. There are, therefore, nine chances in ten that the true value lies between the .05 and .95 curves.

10-04. RELIABILITY OF CORRELATION RESULTS

The reliability of correlation results has been discussed to some extent in section 9. It has been demonstrated that an estimate based on a regression equation has a standard error of estimate expressed by equation 35. The advantage gained in making regression analysis is measured by the determination coefficient or correlation coefficient computed by use of equation 31. There are further reliability criteria that are useful in making decisions in correlation studies. To determine whether it is worthwhile to introduce a particular variable in an analysis, compute the partial correlation coefficient as explained in paragraph 9-07. Even though the partial correlation coefficient is positive, however, it is possible that it is accidentally high and therefore not dependable. The chance that a partial correlation coefficient is accidental (that the true coefficient is zero) can be computed by use of the following equation:

$$\text{Prob}\left[\sqrt{\frac{r^2(\text{df.})}{1-r^2}} > C\right] = \text{Prob}\left[t > C\right] \qquad (44)$$

This equation is also valid when applied to a simple correlation coefficient, where df = N-2.

SECTION 11 - TERMS AND SYMBOLS

TERMS

Analytical method - Method of fitting frequency curves by moments (par 2-05c)
Annual event - The largest (or smallest) event in the year
Cumulative frequency curve - Relation of magnitude to percentage of events exceeding that magnitude (par 2-04)
Deviation - Difference between an individual magnitude and the average for the frequency array (par 4-02a)
Distribution - Function describing the relative frequency with which events of various magnitudes occur (par 2-05c)
Duration curve - Curve indicating the percentage of time that various rates of flow are exceeded at a specified river station (par 2-04e)
Error variance - Square of the standard error (par 9-03d)
Exceedence interval - The average interval of time between values that exceed a specified magnitude; reciprocal of the exceedence frequency
Exceedence frequency - The percentage of values that exceed a specified magnitude (par 2-04a)
Exceedence probability - Probability that an event selected at random will exceed a specified magnitude (par 2-04a)
Expected probability - Statistical average of estimated future probabilities (par 2-05c)
Frequency array - List of events arranged in the order of magnitude (par 2-05b)
Frequency curve - Graphical representation of a frequency distribution, usually with the abscissa as magnitude and the ordinate as relative frequency, but also used interchangeably with cumulative frequency curve.
Geometric mean - Antilogarithm of the average logarithm
Logarithmic normal grid - Grid on which a cumulative frequency curve of a variable whose logarithms are normally distributed will plot as a straight line. (par 3-06)
Logarithmic probability grid - Same as logarithmic normal grid
Maximum likelihood - A designation applied to a statistical estimate that is more likely than any other estimate to be true on the basis of data employed in its estimation (par 4-02c)
Normal distribution - Ideal frequency distribution approached by many observed distributions which are symmetrical and in which the small deviations greatly outnumber the large ones, the possible deviations being virtually unlimited in magnitude by physical conditions.

Parent population - Sum total of events that will occur in the future if pertinent existing controls continue indefinitely (par 2-03)

Partial-duration curve - Cumulative frequency curve of all events above a base value (par 2-04c)

Pearson Type III function - Family of asymmetrical, ideal frequency distributions of which the normal distribution is a special case (ex 39)

Plotting position - Exceedence probability of a magnitude, estimated from its position in a frequency array (par 3-05)

Probability grid - Grid on which a cumulative frequency curve of a normal distributed variable will plot as a straight line (par 3-06)

Random - Condition under which events occur if magnitudes of successive events are not correlated.

Recurrence interval - Exceedence interval, in the case of floods or storms

Return period - Recurrence interval

Standard error - Root-mean-square error (par 9-03)

Standard deviation - Root-mean-square deviation (par 4-02)

Skew coefficient - Function of the third moment of magnitudes about their mean, a measure of asymmetry (par 4-02)

Variance - Mean square deviation; square of the standard deviation

SYMBOLS

a	Regression-equation constant (par 9-02a)
A	Drainage area size
b	Regression-equation coefficient (par 9-02a)
β	Beta coefficient (par 9-07)
C	Any constant
D	Depth of rainfall
df	Degrees of freedom (par 9-03)
g	Skew coefficient (par 4-03c)
k	Coefficient of S (eq 5)
m	Order number of event (par 3-05b)
M	Computed mean
μ	True mean
N	Number of events or number of years of record
P	Probability, specifically exceedence frequency per hundred years
P_∞	Maximum-likelihood exceedence probability (par 4-03d)
Q	Flow or runoff
R	Unadjusted correlation coefficient (par 9-03)
$\bar{R}$	Correlation coefficient, square root of determination coefficient (par 9-03)
r	Partial-correlation coefficient (par 9-07)
S	Computed standard deviation (par 4-03b)
σ	True standard deviation
S_e	Standard error (par 9-03)
T	Duration of runoff
t	A theoretical function (ex 34)
x	Deviation of X from M_X (par 4-02a)
X	Any variable, the dependent variable when subscript is 1, but usually an independent variable; frequently the logarithm of Q
y	Deviation of Y from M_Y (par 4-02a)
Y	Any variable, usually the dependent variable
χ^2	Chi-square, a theoretical variable (ex 35)

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APPENDIX I - KEY QUESTIONS AND ANSWERS

The following questions include general questions that might normally occur to a person after completing this paper and are intended to insure that the main points in the paper have been properly understood.

1. What are the principal advantages of frequency analysis over a simple assumption that a record will repeat itself?

First, a properly conducted frequency study is more accurately representative of future probabilities, and secondly, a frequency study can be used to represent periods of varying length regardless of the period of record and various geographic locations regardless of the location of the record.

2. How can we tell that one method of hydrologic frequency analysis yields more accurate results than another, since it would take hundreds or thousands of years to prove or disprove an estimate?

Reference 24 demonstrates a method of dividing long records into parts so that estimates based on one part are judged by the remaining (unused) portion of the record. When this is done for alternative methods using a great many stations, one is able to select as more dependable the method that yields the best results on the average.

3. Why not design spillways for major dams to pass the 1000-year flood only?

First, the 1000-year flood cannot be estimated dependably--there have already been cases where the 1000-year flood estimate has subsequently been exceeded. Secondly, one structure out of 10 or 20 would fail during their economic lifetime and many more during the expected physical lifetime.

4. If there were 1,000 independent 10-year records in the country, how many should you expect to contain one or more floods larger than the true 100-year magnitude?

About 90

5. What is an objection to using the terms 50-year flood, 100-year flood, etc.?

To some it implies that a 50-year flood will not be exceeded in 50 years whereas it can be exceeded next year. To others it implies that it is a high degree of protection, whereas a structure designed to be safe against a 50-year flood will have a 64 percent probability of failure in a 50-year period, or an 18 percent chance of failure in a 10-year period.

6. Why is the largest flood in a 10-year period not equal to the 10-year flood, on the average?

Because the 10-year flood is defined as one that is exceeded on the average once in 10 years, not equalled. Therefore the 10-year flood should be smaller than the largest flood in a 10-year period, on the average.

7. What are the main factors affecting the accuracy of ordinary frequency determinations?

The length of record and variance of runoff. Compared to the effects of random occurrence, the accuracy of measuring events that do occur is a minor consideration in ordinary cases.

8. Why are extreme-frequency estimates far less reliable than ordinary-frequency estimates?

Because factors that are possible but of rare occurrence such as a rare type of storm, changes in river course, landslide-caused floods, etc., may not be properly reflected in the record.

9. If we were willing to accept an accuracy such that only one case out of 20 would exceed our estimate plus allowable error, how much would our error allowance be for estimating the 2-year and 100-year floods from a 40-year record?

About 20 percent and 40 percent, respectively, where the stream is moderately variable. About 1/3 of these values for stable streams like the Mississippi and far greater for erratic streams.

10. What does a simple correlation coefficient or partial correlation coefficient of 0.5 imply?

That the independent variable being tested reduces the error of estimate by a fraction represented by one minus the square root of one minus the square of the correlation coefficient, or about 13 percent, in this case.

APPENDIX II - PROBLEMS AND SOLUTIONS

The following problems can be used by students as exercises necessary to a thorough understanding of the principles and procedures contained in this paper. Solutions should be read only after the problem has been worked, or to the extent necessary to clarify any misunderstanding.

1. What is the chance that exactly three 50-year floods will occur in a 100-year period? What is the chance that three or more will occur?

This question needs interpretation. Since the 50-year flood is an exact figure, the chance that it will occur exactly is negligible, or zero. We will, however, interpret this question to mean exceeded. The general formula for the exact number of chance events, i, out of N trials is:

$$P = \frac{N!}{i!(N-i)!} p^i(1-p)^{N-i}$$

In which P is the probability of obtaining in N trials exactly i events having a probability p of occurring in a single trial. Substituting 100 for N, 3 for i, and .02 for p, we obtain P=.183, or about one chance in five or six.

To obtain the answer to the second question, we could substitute, 3, 4, 5, etc. up to 100 in the formula and add the probabilities of these mutually exclusive (if one occurs, the others can't) events, or we can shorten the work by substituting 2, 1 and 0 in the formula and subtracting the sum of probabilities from 1.00 (certainty), since it is certain that either (a) 3 or more or (b) 2 or less must occur. Substituting 2, 1 and 0 in the formula, we obtain .274, .271, .271, and .133 in turn for P. One minus their sum is .322, or about one chance in three. It is necessary to know for this solution that zero factorial equals 1.00.

2. What is the chance that a 1000-year flood will be exceeded in the 50-year economic lifetime of a project?

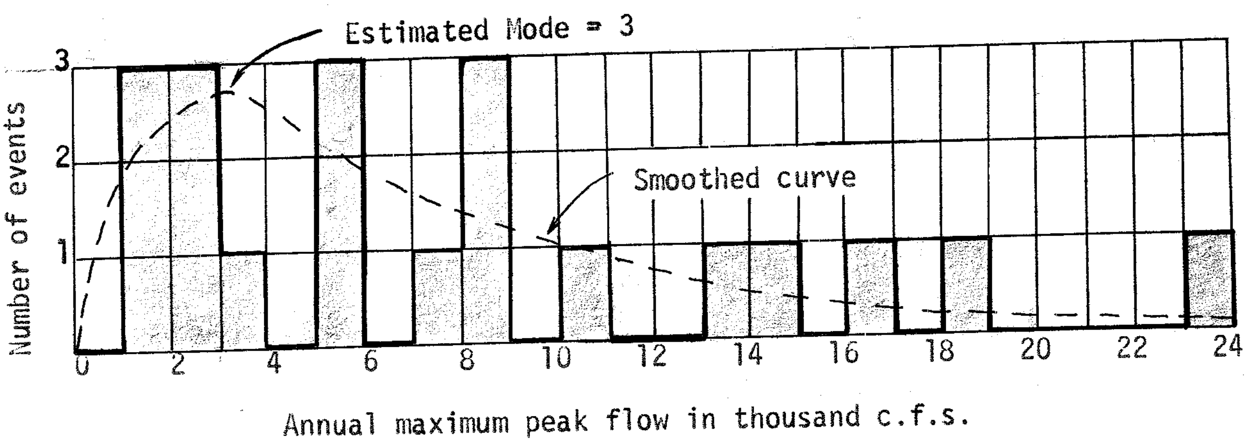
We interpret this to include the chance that the flood will be exceeded more than once, as well as exactly once. We should, therefore, solve for the chance that it will not occur (i=o) and subtract that probability for 1.00, as explained in problem 1. Note that where i = o, the equation resolves to

$$P = (1-p)^N$$

Substituting .001 for p and 50 for N, we obtain .952 for P and, therefore, .048 for the answer, about one chance in 20.

3. Using the peak flows tabulated in problem 5, compute the mean, median, mode, variance, standard deviation, skew coefficient and standard errors of the mean, standard deviation and skew coefficient for the flows and not for the logarithm. What is the chance that the true mean is greater than 6,000 c.f.s.? That the true standard deviation is greater than the 5,000 c.f.s.?

The mean, obtained from equation 2, is 8.04. The median (middle magnitude) is taken as the average of the two middle values of 5.47 and 7.48, since there are an even number of values, and equals 6.48. The mode (most frequent value) should be estimated from a plotting as follows:



The variance, obtained from equation 3, is 41.0. Its square root (S) is the standard deviation, or 6.40. The skew coefficient, obtained from equation 4, is 0.96. The standard error of the mean, obtained from equation 39, is 1.43. The standard error of the standard deviaition is 1.01, from equation 40. The standard error of the skew coefficient, obtained from equation 41, is 0.26.

According to equation 42, the chance that the true mean exceeds 6.00 is equal to the chance that a value of t equal to 2.04 (4.36)/ 6.40 = 1.39 is exceeded in the positive direction. This is 0.91, as determined from exhibit 34. Likewise, according to equation 43, the chance that the true standard deviation is greater than 5,000 c.f.s. is equal to the chance that a value of χ^2 equal to $19(6.40)^2/5.00^2 = 31.1$ is not exceeded. This is 0.96, as determined from exhibit 35.

4. Using the table of random numbers on the following page, generate twenty 5-event random samples from a normal parent population whose mean is 5.00 and standard deviation 1.00. Compute the true exceedence probability of the mean plus 2.18 standard deviations for each sample. Average the 20 probabilities.

This problem is designed to teach speed in analytical computations and to demonstrate the nature of random samples and of statistical inference. First, draw a straight-line frequency curve on arithmetic probability paper with the given mean and standard deviation as demonstrated on page II-5. This represents the parent population. Using the Monte Carlo Method, in order to obtain a random magnitude, enter the curve with an exceedence frequency equaling the random number whose first two digits (including zero) are percentages and remaining digits decimals of a percent. A random number should be selected using a predetermined pattern such as starting with the upper right corner and working down each column. Four digits are sufficient except that a pre-determined procedure is required for obtaining additional digits if the first two or more are zeros or nines.

For each group of 5 random magnitudes, compute the mean and standard deviation using equations 2 and 3. Multiply the computed standard deviation by 2.18 and add to the computed mean to obtain an estimate of the 20-year event. Enter the true frequency curve with this magnitude to obtain its true exceedence frequency. See if the 20 true exceedence frequencies determined from 20 different 5-event random samples average about 5 percent, as exhibit 38 indicates that they should.

5. Using data on page II-6, compute volume frequency curves and plot the curves and data as illustrated on exhibits 15 to 20. Check to make sure that the plotted points and computed curves are reasonably consistent.

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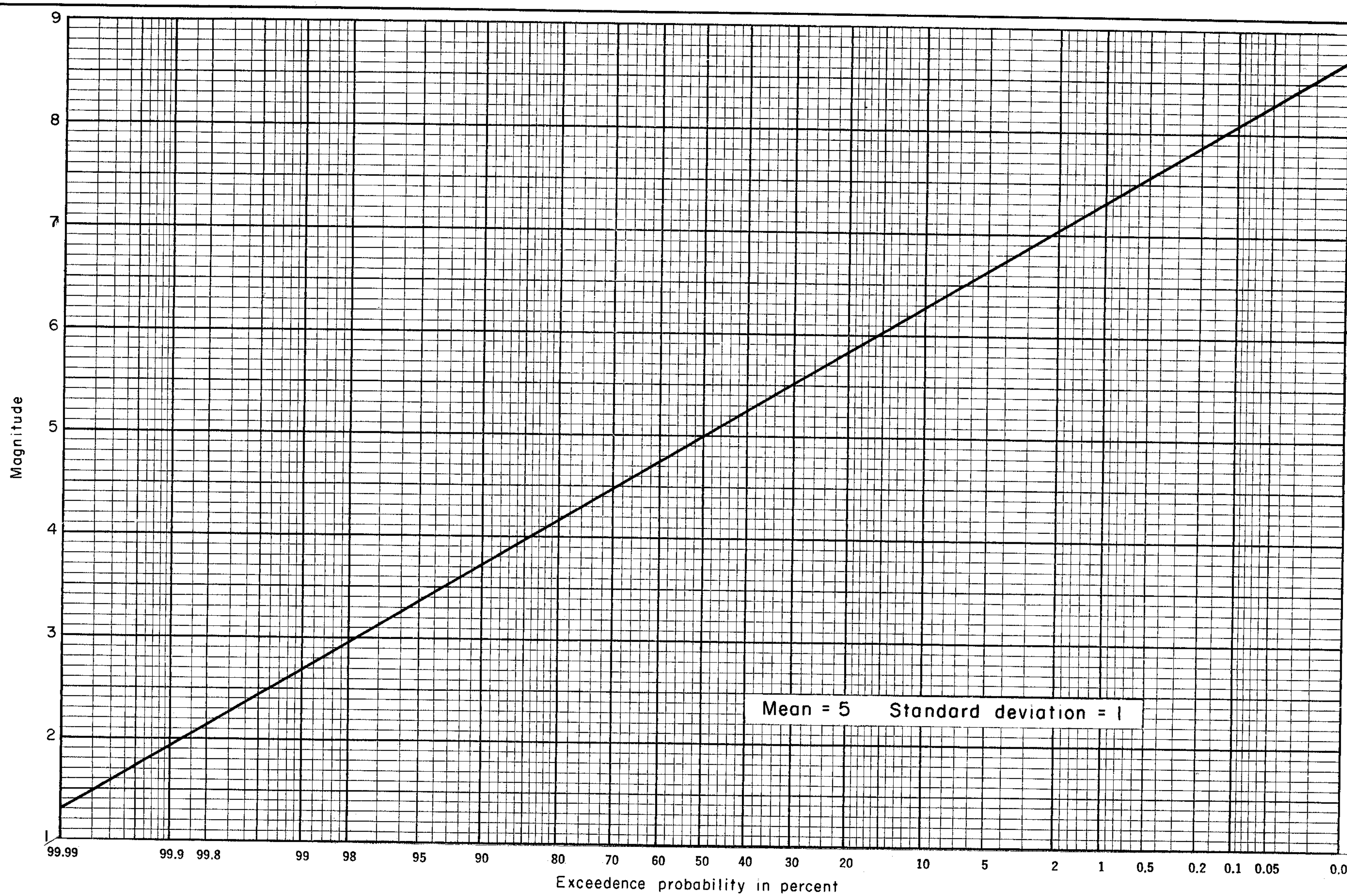
RANDOM NUMBERS

53 74 23 99 67	61 32 28 69 84	94 62 67 86 24	98 33 41 19 95	47 53 53 38 09
63 38 06 86 54	99 00 65 26 94	02 82 90 23 07	79 62 67 80 60	75 91 12 81 19
35 30 58 21 46	06 72 17 10 94	25 21 31 75 96	49 28 24 00 49	55 65 79 78 07
63 43 36 82 69	65 51 18 37 88	61 38 44 12 45	32 92 85 88 65	54 34 81 85 35
98 25 37 55 26	01 91 82 81 46	74 71 12 94 97	24 02 71 37 07	03 92 18 66 75
02 63 21 17 69	71 50 80 89 56	38 15 70 11 48	43 40 45 86 98	00 83 26 91 03
64 55 22 21 82	48 22 28 06 00	61 54 13 43 91	82 78 12 23 29	06 66 24 12 27
85 07 26 13 89	01 10 07 82 04	59 63 69 36 03	69 11 15 83 80	13 29 54 19 28
58 54 16 24 15	51 54 44 82 00	62 61 65 04 69	38 18 65 18 97	85 72 13 49 21
34 85 27 84 87	61 48 64 56 26	90 18 48 13 26	37 70 15 42 57	65 65 80 39 07
03 92 18 27 46	57 99 16 96 56	30 33 72 85 22	84 64 38 56 98	99 01 30 98 64
62 95 30 27 59	37 75 41 66 48	86 97 80 61 45	23 53 04 01 63	45 76 08 64 27
08 45 93 15 22	60 21 75 46 91	98 77 27 85 42	28 88 61 08 84	69 62 03 42 73
07 08 55 18 40	45 44 75 13 90	24 94 96 61 02	57 55 66 83 15	73 42 37 11 61
01 85 89 95 66	51 10 19 34 88	15 84 97 19 75	12 76 39 43 78	64 63 91 08 25
72 84 71 14 35	19 11 58 49 26	50 11 17 17 76	86 31 57 20 18	95 60 78 46 75
88 78 28 16 84	13 52 53 94 53	75 45 69 30 96	73 89 65 70 31	99 17 43 48 76
45 17 75 65 57	28 40 19 72 12	25 12 74 75 67	60 40 60 81 19	24 62 01 61 16
96 76 28 12 54	22 01 11 94 25	71 96 16 16 88	68 64 36 74 45	19 59 50 88 92
43 31 67 72 30	24 02 94 08 63	38 32 36 66 02	69 36 38 25 39	48 03 45 15 22
50 44 66 44 21	66 06 58 05 62	68 15 54 35 02	42 35 48 96 32	14 52 41 52 48
22 66 22 15 86	26 63 75 41 99	58 42 36 72 24	58 37 52 18 51	03 37 18 39 11
96 24 40 14 51	23 22 30 88 57	95 67 47 29 83	94 69 40 06 07	18 16 36 78 86
31 73 91 61 19	60 20 72 93 48	98 57 07 23 69	65 95 39 69 58	56 80 30 19 44
78 60 73 99 84	43 89 94 36 45	56 69 47 07 41	90 22 91 07 12	78 35 34 08 72
84 37 90 61 56	70 10 23 98 05	85 11 34 76 60	76 48 45 34 60	01 64 18 39 96
36 67 10 08 23	98 93 35 08 86	99 29 76 29 81	33 34 91 58 93	63 14 52 32 52
07 28 59 07 48	89 64 58 89 75	83 85 62 27 89	30 14 78 56 27	86 63 59 80 02
10 15 83 87 60	79 24 31 66 56	21 48 24 06 93	91 98 94 05 49	01 47 59 38 00
55 19 68 97 65	03 73 52 16 56	00 53 55 90 27	33 42 29 38 87	22 13 88 83 34
53 81 29 13 39	35 01 20 71 34	62 33 74 82 14	53 73 19 09 03	56 54 29 56 93
51 86 32 68 92	33 98 74 66 99	40 14 71 94 58	45 94 19 38 81	14 44 99 81 07
35 91 70 29 13	80 03 54 07 27	96 94 78 32 66	50 95 52 74 33	13 80 55 62 54
37 71 67 95 13	20 02 44 95 94	64 85 04 05 72	01 32 90 76 14	53 89 74 60 41
93 66 13 83 27	92 79 64 64 72	28 54 96 53 84	48 14 52 98 94	56 07 93 89 30
02 96 08 45 65	13 05 00 41 84	93 07 54 72 59	21 45 57 09 77	19 48 56 27 44
49 83 43 48 35	82 88 33 69 96	72 36 04 19 76	47 45 15 18 60	82 11 08 95 97
84 60 71 62 46	40 80 81 30 37	34 39 23 05 38	25 15 35 71 30	88 12 57 21 77
18 17 30 88 71	44 91 14 88 47	89 23 30 63 15	56 34 20 47 89	99 82 93 24 98
79 69 10 61 78	71 32 76 95 62	87 00 22 58 40	92 54 01 75 25	43 11 71 99 31
75 93 36 57 83	56 20 14 82 11	74 21 97 90 65	96 42 68 63 86	74 54 13 26 94
38 30 92 29 03	06 28 81 39 38	62 25 06 84 63	61 29 08 93 67	04 32 92 08 09
51 29 50 10 34	31 57 75 95 80	51 97 02 74 77	76 15 48 49 44	18 55 63 77 09
21 31 38 86 24	37 79 81 53 74	73 24 16 10 33	52 83 90 94 76	70 47 14 54 36
29 01 23 87 88	58 02 39 37 67	42 10 14 20 92	16 55 23 42 45	54 96 09 11 06
95 33 95 22 00	18 74 72 00 18	38 79 58 69 32	81 76 80 26 92	82 80 84 25 39
90 84 60 79 80	24 36 59 87 38	82 07 53 89 35	96 35 23 79 18	05 98 90 07 35
46 40 62 98 82	54 97 20 56 95	15 74 80 08 32	16 46 70 50 80	67 72 16 42 79
20 31 89 03 43	38 46 82 68 72	32 14 82 99 70	80 60 47 18 97	63 49 30 21 30
71 59 73 05 50	08 22 23 71 77	91 01 93 20 49	82 96 59 26 94	66 39 67 98 60



HYPOTHETICAL NORMAL POPULATION

Average flows in thousand c.f.s.

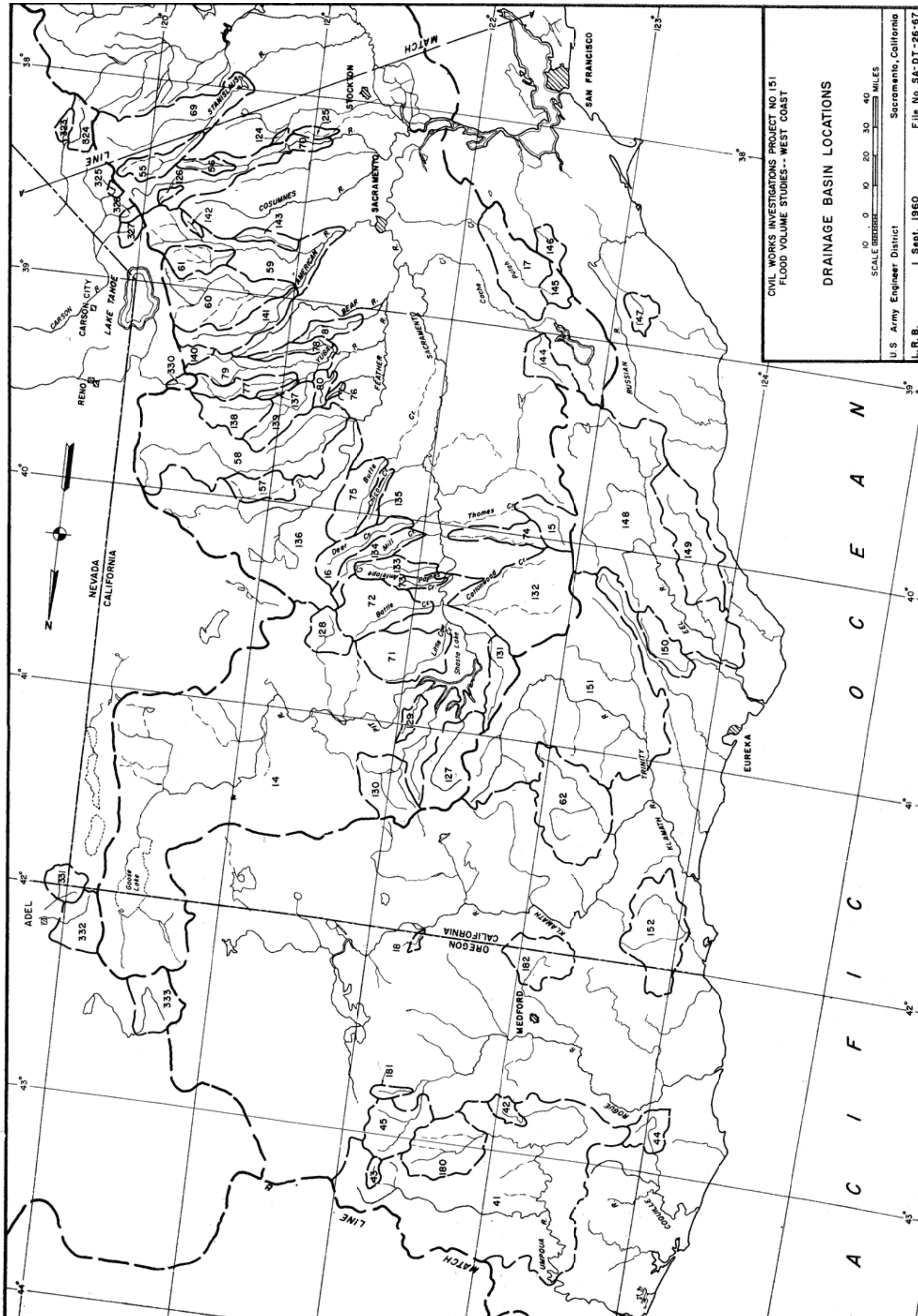
Year	Peak	1-day	3-day	10-day	30-day	90-day	1 year
1936-37	2.18	1.94	1.79	1.13	0.71	0.54	0.16
1937-38	16.5	9.70	7.40	2.84	1.58	1.42	0.62
-39	1.55	1.04	0.63	0.60	0.42	0.22	.089
-40	14.5	10.8	7.80	3.40	1.77	1.21	0.39
-41	13.2	7.86	6.20	3.05	1.64	1.44	0.60
-42	8.12	4.65	3.43	2.17	1.43	0.99	0.40
-43	18.6	10.2	5.93	2.42	1.19	0.78	0.28
-44	1.16	0.82	0.64	0.44	0.31	0.25	.092
-45	2.31	1.48	1.23	0.98	0.59	0.39	0.17
-46	8.99	5.76	5.30	2.67	1.41	0.67	0.28
-47	5.21	3.20	1.66	0.73	0.46	0.33	0.11
-48	5.47	3.39	2.66	1.38	0.67	0.47	0.19
-49	1.66	1.48	1.33	0.91	0.79	0.59	0.18
1949-50	3.04	2.13	1.51	1.13	0.76	0.52	0.18
-51	7.48	4.39	3.40	1.96	1.40	0.76	0.30
-52	8.86	5.36	3.63	2.00	1.36	1.06	0.44
-53	10.8	7.75	4.17	2.95	1.75	0.84	0.35
-54	5.39	4.21	2.71	1.56	1.23	0.89	0.33
-55	2.19	1.50	0.72	0.56	0.39	0.25	0.13
-56	23.5	16.3	11.1	5.47	2.89	1.52	0.60

6. Using the data on the following page, compute the multiple linear regression equation, determination coefficient, standard error of estimate, and the partial correlation coefficient of the variable having the smallest beta coefficient. Using the regression equation, compute the residual (regression constant) for each station, plot in the drainage area on the following map, and draw lines of equal residual.

Extensions required for computing the regression equation are computed as illustrated on exhibit 18. The equation coefficients and constant are computed most easily by use of the Crout's Method (paragraph 9-02c). The equation should be:

CORRELATION DATA

Station number	Mean log of peak flow in c.f.s.	Log of drainage area in sq. mi.	Log of normal annual precip. in inches	Elev. above snowline in hundred ft.
14	5.08	3.97	1.71	54
15	3.72	2.27	1.62	55
16	3.71	2.30	1.66	54
17	4.41	2.76	1.58	17
18	2.23	1.30	1.58	49
41	5.00	3.57	1.72	46
42	3.38	1.89	1.60	49
43	2.46	1.62	1.79	71
44	4.18	2.23	1.94	42
45	3.64	2.52	1.81	68
55	3.19	2.21	1.74	72
56	2.83	1.83	1.64	46
57	3.55	2.26	1.67	59
58	4.27	3.13	1.60	61
59	4.61	3.28	1.72	55
60	4.17	2.79	1.80	57
61	3.43	2.25	1.77	63
62	4.20	2.87	1.81	55
69	3.98	2.95	1.67	58
70	3.87	2.80	1.64	54
71	4.27	2.63	1.65	34
72	3.78	2.56	1.63	51
73	3.45	1.97	1.50	27
74	3.61	2.15	1.51	36
75	3.72	2.17	1.78	46
76	3.19	1.48	1.60	22
77	3.81	2.30	1.77	54
78	3.58	1.77	1.64	20
79	4.51	3.08	1.78	54
80	3.61	1.85	1.71	27
81	3.48	2.00	1.51	17
124	2.89	1.31	1.38	12
125	2.75	1.68	1.28	11
126	2.81	1.36	1.78	81
127	4.21	2.63	1.80	68
128	2.44	2.09	1.62	68
129	3.73	1.81	1.86	44
130	3.25	2.58	1.75	73
131	3.79	2.36	1.75	42
132	4.26	2.98	1.65	22
133	3.61	2.09	1.59	29
134	3.64	2.13	1.67	37
135	3.49	1.83	1.72	28
136	4.70	3.56	1.68	61
137	3.21	1.54	1.79	43
138	3.89	2.39	1.79	67
139	4.22	2.68	1.81	55
140	3.31	1.71	1.70	77
141	4.06	2.54	1.76	48
142	2.46	1.36	1.69	65
143	3.56	2.00	1.54	24
144	3.76	2.30	1.61	35
145	3.96	2.05	1.73	18
146	3.64	1.91	1.61	16
147	3.85	1.94	1.74	16
148	5.16	3.49	1.76	36
149	4.62	2.73	1.85	21
150	4.23	2.30	1.86	42
151	4.70	3.45	1.76	52
152	4.85	2.79	1.96	37
180	4.23	2.65	1.77	51
181	2.81	1.60	1.81	72
182	3.69	2.47	1.59	45
323	2.12	1.80	1.41	87
324	2.48	2.26	1.49	92
325	2.13	1.30	1.54	89
326	1.73	1.15	1.57	90
327	2.32	1.82	1.51	90
330	2.49	1.52	1.58	75
331	3.03	2.29	1.08	72
332	2.93	2.40	1.15	76
333	2.84	2.44	1.32	73


CIVIL WORKS INVESTIGATIONS PROJECT NO 151
FLOOD VOLUME STUDIES-- WEST COAST
DRAINAGE BASIN LOCATIONS
SCALE
MILES
U.S. Army Engineer District
Sacramento, California
L.R.B.
1 Sept. 1960
File No. SA-DT-26-67
P A C I F I C O C E A N
NEVADA
CALIFORNIA
OREGON
CALIFORNIA
MATCH LINE
SAN FRANCISCO
STOCKTON
SACRAMENTO
CARSON CITY
LAKE TAHOE
RENO
EUREKA
MEDFORD
ADEL

$$X_1 = 0.945 X_2 + 1.37 X_3 - 0.014 X_4 - 0.147$$

The determination coefficient, computed by use of equations 31 and 32 is 0.925. The standard error of estimate, from equation 35, is 0.21. Beta coefficients, from equation 38, are 0.75, 0.29, and -0.38 in turn. In order to obtain the partial correlation coefficient for the second variable, it is necessary to repeat Crout's Method using all but the second variable. The new determination coefficient is 0.846, and the partial correlation coefficient, from equation 37, is 0.51. The residual for each station is computed by transposing the regression equation as follows and substituting observed values of X_2, X_3 and X_4 for each station in turn:

$$C = X_1 - 0.945 X_2 - 1.37 X_3 + 0.014 X_4$$

7. Adjust the statistics for 1-day flows in problem 5 to take advantage of the longer record on exhibit 15 and the further adjustment of statistics for that station on exhibit 18.

We note on exhibit 15 that there are an additional 10 years of 1-day flow record on Mill Creek that are not contained in problem 5. Also, the Mill Creek statistics were adjusted by use of Feather River flows recorded over a 47-year period (exhibit 18) to obtain the equivalent of an additional 13 years, or 43 years in all.

By pairing the 20 years of concurrent record, the determination coefficient is found by use of equations 7 and 8 to be 0.70, and other statistics are simultaneously computed as illustrated for Mill Creek on exhibit 18. These turn out to be:

	20 years	43 years
Mill Creek mean log	3.474	3.410
Problem 5 " "	3.578	
Mill Creek std dev	.317	.287
Problem 5 " "	.372	

By use of equations 9 to 11, adjusted statistics and the length of equivalent record are found to be:

Mean log	3.524
Standard deviation	.347
Equivalent record	36

8. Using exhibits 24 to 27, compute a regionalized estimate of logarithmic mean and standard deviation of peak flows for Mill Creek. Compare these with the computed values on exhibit 18 and select a value for each that you would recommend for adoption. Required data are as follows:

Drainage area	134 sq. mi.
Basin normal annual precip.	47 in.
Average basin elevation	2900 ft. m.s.l.

This example is already worked out in paragraph 7-12. The standard deviation in hundredths is a basin-mean value estimated from exhibit 25 and is 0.31. The basin-mean value of C_p determined from exhibit 24 for entry into equation 11 of reference 21, also given in paragraph 7-12, is 42. A value of K for use in that equation, obtained from exhibit 26, is 0.71. The computed Q is 4220 c.f.s. and the corresponding logarithm is 3.625. This compares to 3.661 on exhibit 18, and the regional standard deviation of 0.31 compares to 0.281 on exhibit 18. Considering that the equivalent length of record on exhibit 18 is 43 years, the standard error of the calculated mean from exhibit 23 is .152 (0.281) or 0.43 and the standard error of the calculated standard deviation is antilog .047, or a factor of 1.11. Since the regional values are less than one standard error from the computed values, and since computed values can easily be off by one standard error, it is best to adopt the regional values for the sake of consistency with adjoining basins.

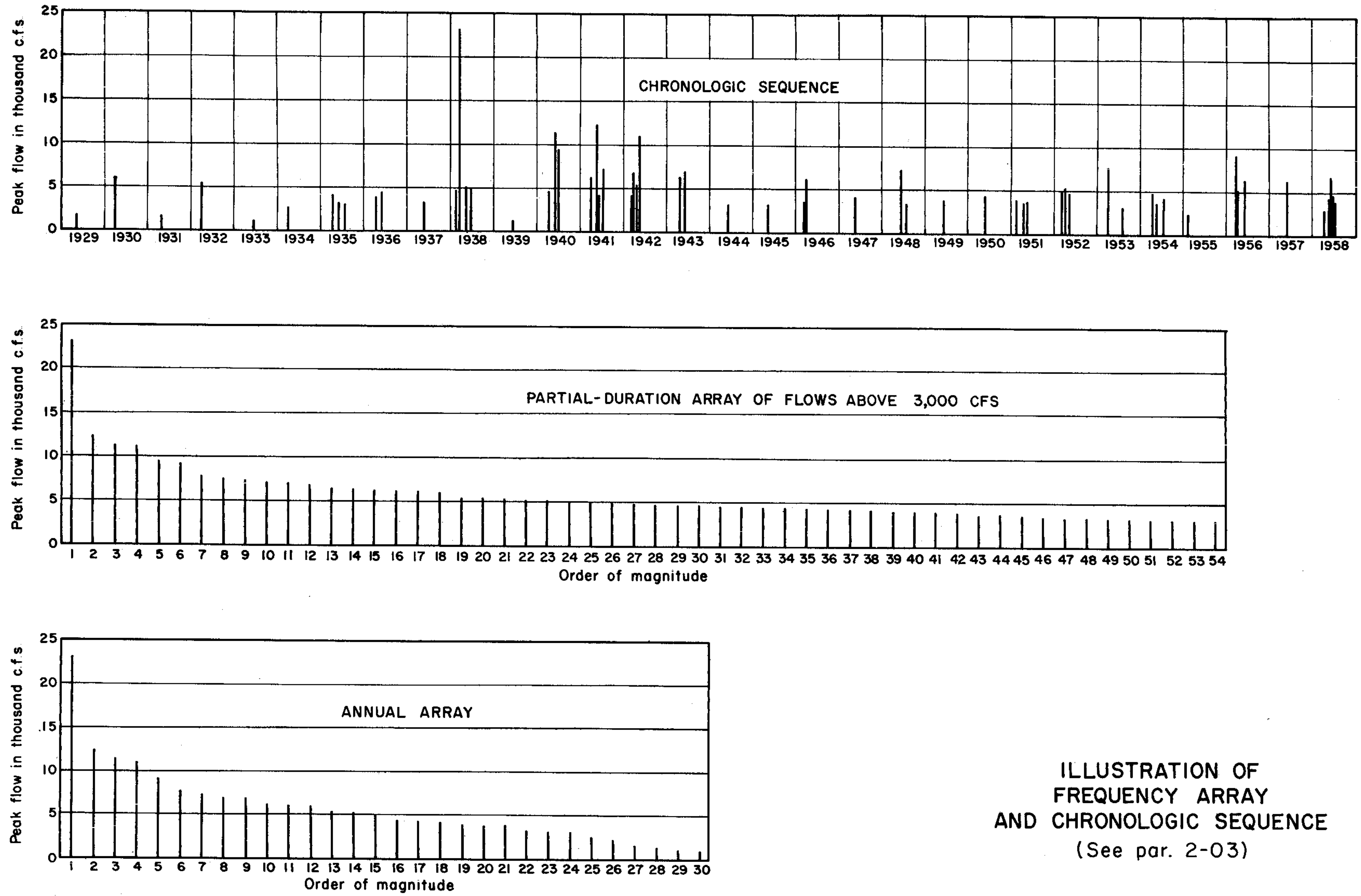
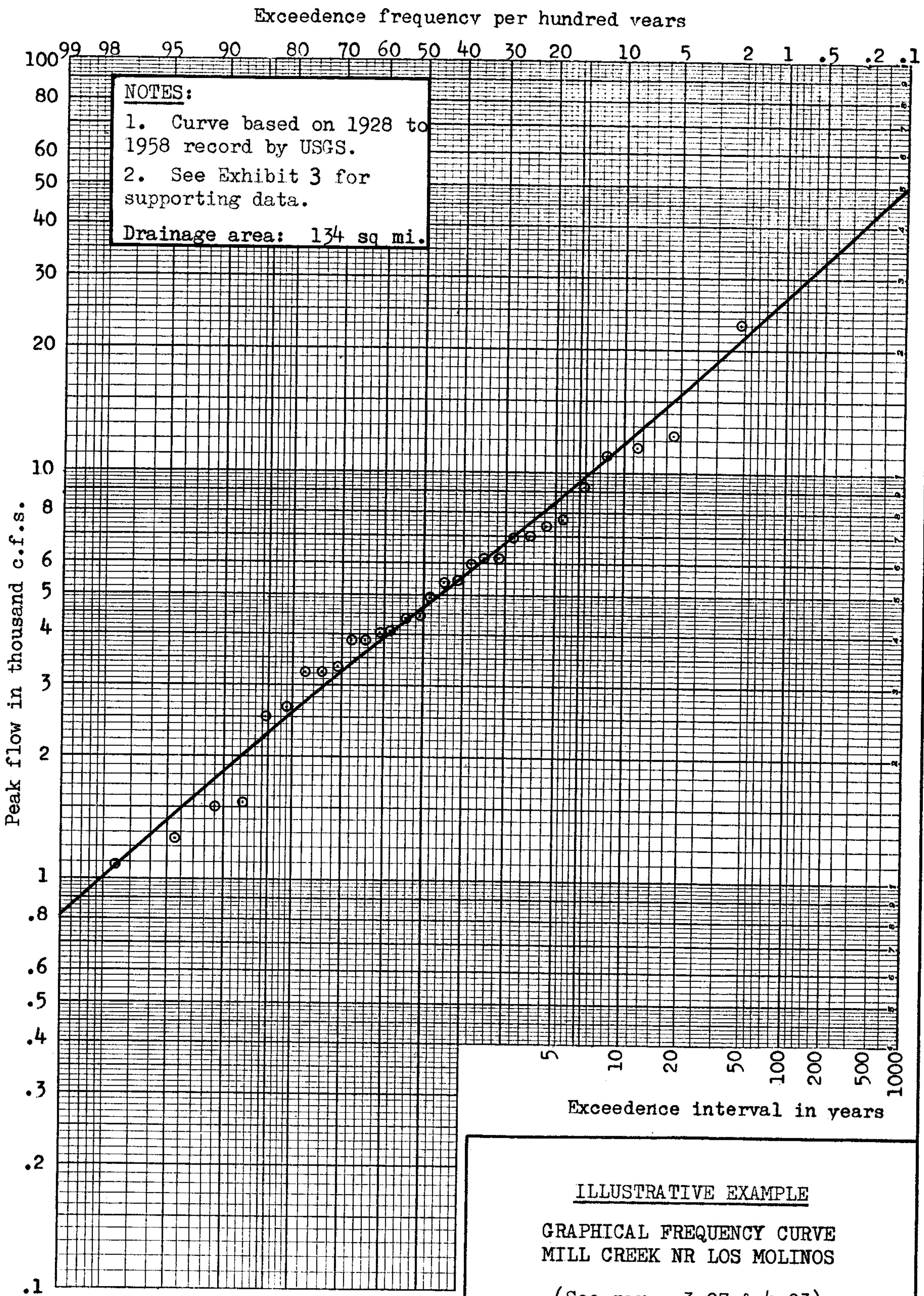


ILLUSTRATION OF
FREQUENCY ARRAY
AND CHRONOLOGIC SEQUENCE
(See par. 2-03)

EXHIBIT I



ILLUSTRATIVE EXAMPLE

GRAPHICAL FREQUENCY CURVE
MILL CREEK NR LOS MOLINOS

(See pars. 3-07 & 4-03)

ILLUSTRATIVE EXAMPLE

TABULATION OF PEAK FLOW FREQUENCY DATA
MILL CREEK NEAR LOS MOLINOS, CALIFORNIA

(See sec. 3)

Events in order recorded: Annual maximum, and all other flows above 3000 c.f.s. and separated by 10 days or more from larger flows						Events in decreasing order of magnitude: Annual Maximum		Partial-duration series (flows above 3000 c.f.s.)			
Water Year (1)	Date (2)	Flow (cfs) (3)	Water Year (4)	Date (5)	Flow (cfs) (6)	Plotting Position (7)	Flow (cfs) (8)	Plotting Position (9)	Flow (cfs) (10)	Plotting Position (11)	Flow (cfs) (12)
1928-29	3 Feb	1,520	1943-44	4 Mar	3,220	2.3	23,000	2.3	23,000	91.7	4,650
1929-30	15 Dec	6,000	1944-45	5 Feb	3,230	5.6	12,200	5.6	12,200	95.0	4,600
1930-31	23 Jan	1,500	1945-46	4 Dec	3,660	8.9	11,400	8.9	11,400	98.3	4,540
1931-32	24 Dec	5,440	1945-46	21 Dec	6,180	12.2	11,000	12.2	11,000	101.7	4,430
1932-33	16 Mar	1,080	1946-47	12 Feb	4,070	15.4	9,180	15.4	9,360	105.0	4,380
1933-34	29 Dec	2,630	1947-48	23 Mar	7,320	18.7	7,710	18.7	9,180	108.3	4,330
1934-35	4 Jan	4,010	1947-48	28 Apr	3,380	22.0	7,320	22.0	7,710	111.7	4,250
1934-35	28 Feb	3,190	1948-49	11 Mar	3,870	25.3	6,970	25.3	7,320	115.0	4,240
1934-35	8 Apr	3,040	1949-50	4 Feb	4,430	28.6	6,880	28.6	7,260	118.3	4,130
1935-36	11 Jan	3,930	1950-51	16 Nov	3,870	31.9	6,180	31.9	6,970	121.7	4,070
1935-36	21 Feb	4,380	1950-51	22 Jan	3,510	35.2	6,140	35.2	6,910	125.0	4,010
1936-37	14 Feb	3,310	1950-51	11 Feb	3,660	38.5	6,000	38.5	6,880	128.3	3,970
1937-38	20 Nov	4,700	1951-52	1 Dec	4,930	41.8	5,440	41.8	6,480	131.7	3,930
1937-38	11 Dec	23,000	1951-52	26 Dec	5,280	45.1	5,280	45.1	6,450	135.0	3,870
1937-38	2 Feb	5,050	1951-52	1 Feb	4,650	48.4	4,910	48.4	6,240	138.3	3,870
1937-38	23 Mar	4,950	1952-53	9 Jan	7,710	51.6	4,430	51.7	6,180	141.7	3,660
1938-39	8 Mar	1,260	1952-53	27 Apr	3,070	54.9	4,380	55.0	6,140	145.0	3,660
1939-40	2 Jan	4,600	1953-54	17 Jan	4,910	58.2	4,070	58.3	6,000	148.3	3,510
1939-40	28 Feb	11,400	1953-54	13 Feb	3,300	61.5	4,010	61.7	5,450	151.7	3,380
1939-40	30 Mar	9,360	1953-54	4 Apr	4,240	64.8	3,870	65.0	5,440	155.0	3,310
1940-41	24 Dec	6,240	1954-55	11 Nov	2,480	68.1	3,870	68.3	5,280	158.3	3,300
1940-41	10 Feb	12,200	1955-56	22 Dec	9,180	71.4	3,310	71.7	5,050	161.7	3,230
1940-41	1 Mar	4,250	1955-56	7 Jan	5,020	74.7	3,230	75.0	5,020	165.0	3,220
1940-41	4 Apr	7,260	1955-56	22 Feb	6,480	78.0	3,220	78.3	4,950	168.3	3,190
1941-42	3 Dec	4,130	1956-57	24 Feb	6,140	81.3	2,630	81.7	4,930	171.7	3,070
1941-42	16 Dec	6,910	1957-58	26 Jan	3,060	84.6	2,480	85.0	4,910	175.0	3,060
1941-42	27 Jan	5,450	1957-58	12 Feb	4,330	87.8	1,520	88.3	4,700	178.3	3,040
1941-42	6 Feb	11,000	1957-58	24 Feb	6,880	91.1	1,500				
1942-43	21 Jan	6,450	1957-58	21 Mar	4,540	94.4	1,260				
1942-43	8 Mar	6,970	1957-58	2 Apr	3,970	97.7	1,080				

EXHIBIT 3

ILLUSTRATIVE EXAMPLE

ANALYTICAL COMPUTATION OF PEAK-FLOW FREQUENCY CURVE
MILL CREEK NEAR LOS MOLINOS, CALIFORNIA

(See par. 4-03)

(1) Water Year	(2) Flow (c.f.s.)	(3) Log (X)	(4) Dev. (x)	(5) x^2
1928-29	1,520	3.18	-.49	.240
30	6,000	3.78	.11	.012
31	1,500	3.18	-.49	.240
32	5,440	3.74	.07	.005
33	1,080	3.03	-.64	.410
34	2,630	3.42	-.25	.062
1934-35	4,010	3.60	-.07	.005
36	4,380	3.64	-.03	.001
37	3,310	3.52	-.15	.022
38	23,000	4.36	.69	.476
39	1,260	3.10	-.57	.325
1939-40	11,400	4.06	.39	.152
41	12,200	4.09	.42	.176
42	11,000	4.04	.37	.137
43	6,970	3.84	.17	.029
44	3,220	3.51	-.16	.026
1944-45	3,230	3.51	-.16	.026
46	6,180	3.79	.12	.014
47	4,070	3.61	-.06	.004
48	7,320	3.86	.19	.036
49	3,870	3.59	-.08	.006
1949-50	4,430	3.65	-.02	.000
51	3,870	3.59	-.08	.006
52	5,280	3.72	.05	.002
53	7,710	3.89	.22	.048
54	4,910	3.69	.02	.000
1954-55	2,480	3.39	-.28	.078
56	9,180	3.96	.29	.084
57	6,140	3.79	.12	.014
58	6,880	3.84	.17	.029
	(6) N	30		
	(7) ΣX	109.97	-.13	2.665
	(8) M	3.666		
	(9) ΣX^2	405.7813	$S^2 = 2.6679/29 = .091996$	
	(10) $(\Sigma X)^2/N$	403.1134		
	(11) Σx^2	2.6679	S = .303	

(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
(20) P_n	0.25	1.0	10	50	90	99	99.75
(21) k (Ex 38)	3.09	2.50	1.33	0	-1.33	-2.50	-3.09
(22) Log Q (Eq 5)	4.602	4.424	4.069	3.666	3.263	2.908	2.730
(23) Q, cfs	40,000	26,600	11,700	4,630	1,830	809	537

Note: Columns 4 and 5 are not required when desk calculator is available, but are shown to illustrate procedure usable without desk calculator. The difference between 2.665 and 2.668 is due to rounding values in column 4.

ANALYTICAL FREQUENCY COMPUTATION AND ADJUSTMENT

(See sec. 5)

I Computation of Statistics

Water Year	Annual max flow(Q) Sta. 1 cfs	Sta. 2 cfs	Log Q Sta. 1 (X)	Sta. 2 (Y)	Sta. 2
(1)	(2)	(3)	(4)	(5)	(6)
1911-12		4,570			3.66
1912-13		7,760			3.89
1913-14		32,400			4.51
1914-15		27,500			4.44
1915-16		19,000			4.28
1916-17		24,000			4.38
1917-18		13,200			4.12
1918-19		15,500			4.19
1919-20		10,200			4.01
1920-21		14,100			4.15
1921-22		14,800			4.17
1922-23		10,500			4.02
1923-24		11,500			4.06
1924-25		27,500			4.44
1925-26		17,800			4.25
1926-27		36,300			4.56
1927-28		67,600			4.83
1928-29	1,520	5,500	3.18	3.74	(Values same as in Column 5)
1929-30	6,000	25,500	3.78	4.41	
1930-31	1,500	5,570	3.18	3.75	
1931-32	5,440	9,980	3.74	4.00	
1932-33	1,080	5,100	3.03	3.71	
1933-34	2,630	11,100	3.42	4.05	
1934-35	4,010	25,500	3.60	4.41	
1935-36	4,380	38,200	3.64	4.58	
1936-37	3,310	7,920	3.52	3.90	
1937-38	23,000	93,000	4.36	4.97	
1938-39	1,260	3,230	3.10	3.51	
1939-40	11,400	60,200	4.06	4.78	
1940-41	12,200	30,300	4.09	4.48	
1941-42	11,000	35,100	4.04	4.55	
1942-43	6,970	54,300	3.84	4.73	
1943-44	3,220	8,460	3.51	3.93	
1944-45	3,230	28,600	3.51	4.46	
1945-46	6,180	22,000	3.79	4.34	
1946-47	4,070	17,800	3.61	4.25	
1947-48	7,320	16,600	3.86	4.22	
1948-49	3,870	6,140	3.59	3.79	
1949-50	4,430	17,900	3.65	4.25	
1950-51	3,870	50,200	3.59	4.70	
1951-52	5,280	21,000	3.72	4.32	
1952-53	7,710	40,000	3.89	4.60	
1953-54	4,910	22,900	3.69	4.36	
1954-55	2,480	5,900	3.39	3.77	
1955-56	9,180	104,000	3.96	5.02	
1956-57	6,140	32,700	3.79	4.51	
1957-58	6,880	39,300	3.84	4.59	

		(10)	(11)	(12)
	N	30	30	47
	ΣX	109.97	128.68	200.64
(Eq. 2)	M	3.666	4.289	4.269
		(7)	(8)	(9)
	ΣX^2	405.7813	556.5276	862.3764
	$(\Sigma X)^2/N$	403.1134	551.9514	856.5194
	Σx^2	2.6679	4.5762	5.8570
	$\Sigma x^2/N - 1$	.0920	.1578	.1273
(Eq. 3)	S	.303	.397	.357
	ΣXY	474.5925		
	ΣXΣY/N	471.6980		
	Σxy	2.8945		

II Adjustment of Statistics

(7) $S_1 = .303$ (Eq. 3)

(8) $S_2 = .397$ (Eq. 3)

(9) $S_2' = .357$ (Eq. 3)

(10) $M_1 = 3.666$ (Eq. 2)

(11) $M_2 = 4.289$ (Eq. 2)

(12) $M_2' = 4.269$ (Eq. 2)

(13) $R^2 = \dfrac{(2.8945)^2}{(2.6679)(4.5762)} = .685$ (Eq. 8)

(14) $\bar{R}^2 = 1 - (1 - .685)\dfrac{29}{28} = .67$ (Eq. 7)

(15) $S_1' = .303 + (.357 - .397)(.685)(.303/.397)$
$= .282$ (Eq. 9)

(16) $M_1' = 3.666 + (4.269 - 4.289)(.685)^{\frac{1}{2}}(.303/.397)$
$= 3.653$ (Eq. 10)

(17) $g = .00$ (Paragraph 4-03c)

(18) $N_1' = \dfrac{30}{1 - \frac{17}{47}(.67)} = 39.6$ years (Eq. 11)

III Computation of frequency curve based on M_1', S_1' and g

$P_{N'}$	k (Ex. 38)	Log Q ($M_1' + kS_1'$)	Q
(19)	(20)	(21)	(22)
.25	3.01	4.501	31,700
1.0	2.45	4.344	22,100
5.0	1.70	4.132	13,600
10.0	1.32	4.025	10,600
30.0	0.54	3.805	6,380
50.0	0.00	3.653	4,500
70.0	-0.54	3.501	3,170
90.0	-1.32	3.281	1,910
95.0	-1.70	3.174	1,490
99.0	-2.45	2.962	916
99.75	-3.01	2.804	637

ERRORS OF ESTIMATED VALUES

As Coefficients of Standard Deviation

(See par. 10-03)

Level of Significance*	Years of Record (N)	Exceedence Frequency in Percent 0.1	1	10	50	90	99	99.9
.05	5	4.41	3.41	2.12	.95	.76	1.00	1.22
	10	2.11	1.65	1.07	.58	.57	.76	.94
	15	1.52	1.19	.79	.46	.48	.65	.80
	20	1.23	.97	.64	.39	.42	.58	.71
	30	.93	.74	.50	.31	.35	.49	.60
	40	.77	.61	.42	.27	.31	.43	.53
	50	.67	.54	.36	.24	.28	.39	.49
	70	.55	.44	.30	.20	.24	.34	.42
	100	.45	.36	.25	.17	.21	.29	.37
.25	5	1.41	1.09	.68	.33	.31	.41	.49
	10	.77	.60	.39	.22	.24	.32	.39
	15	.57	.45	.29	.18	.20	.27	.34
	20	.47	.37	.25	.15	.18	.24	.30
	30	.36	.29	.19	.12	.15	.20	.25
	40	.30	.24	.16	.11	.13	.18	.22
	50	.27	.21	.14	.10	.12	.16	.20
	70	.22	.17	.12	.08	.10	.14	.18
	100	.18	.14	.10	.07	.09	.12	.15
.75	5	- .49	- .41	- .31	-.33	- .68	-1.09	-1.41
	10	- .39	- .32	- .24	-.22	- .39	- .60	- .77
	15	- .34	- .27	- .20	-.18	- .29	- .45	- .57
	20	- .30	- .24	- .18	-.15	- .25	- .37	- .47
	30	- .25	- .20	- .15	-.12	- .19	- .29	- .36
	40	- .22	- .18	- .13	-.11	- .16	- .24	- .30
	50	- .20	- .16	- .12	-.10	- .14	- .21	- .27
	70	- .18	- .14	- .10	-.08	- .12	- .17	- .22
	100	- .15	- .12	- .09	-.07	- .10	- .14	- .18
.95	5	-1.22	-1.00	- .76	-.95	-2.12	-3.41	-4.41
	10	- .94	- .76	- .57	-.58	-1.07	-1.65	-2.11
	15	- .80	- .65	- .48	-.46	- .79	-1.19	-1.52
	20	- .71	- .58	- .42	-.39	- .64	- .97	-1.23
	30	- .60	- .49	- .35	-.31	- .50	- .74	- .93
	40	- .53	- .43	- .31	-.27	- .42	- .61	- .77
	50	- .49	- .39	- .28	-.24	- .36	- .54	- .67
	70	- .42	- .34	- .24	-.20	- .30	- .44	- .55
	100	- .37	- .29	- .21	-.17	- .25	- .36	- .45

* Chance of true value being greater than sum of normal-curve value and given error.

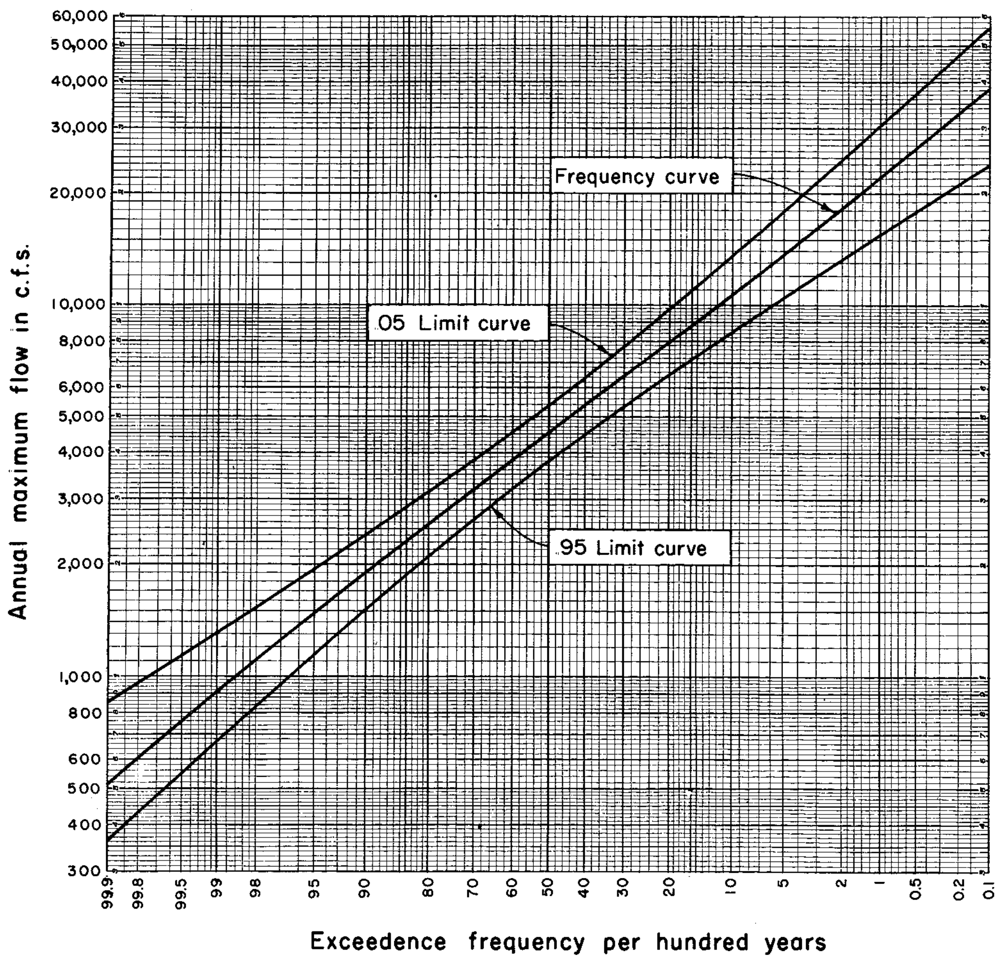
LIMIT-CURVE COMPUTATION

(See par. 10-03)

(Based on equivalent of 39-year record)

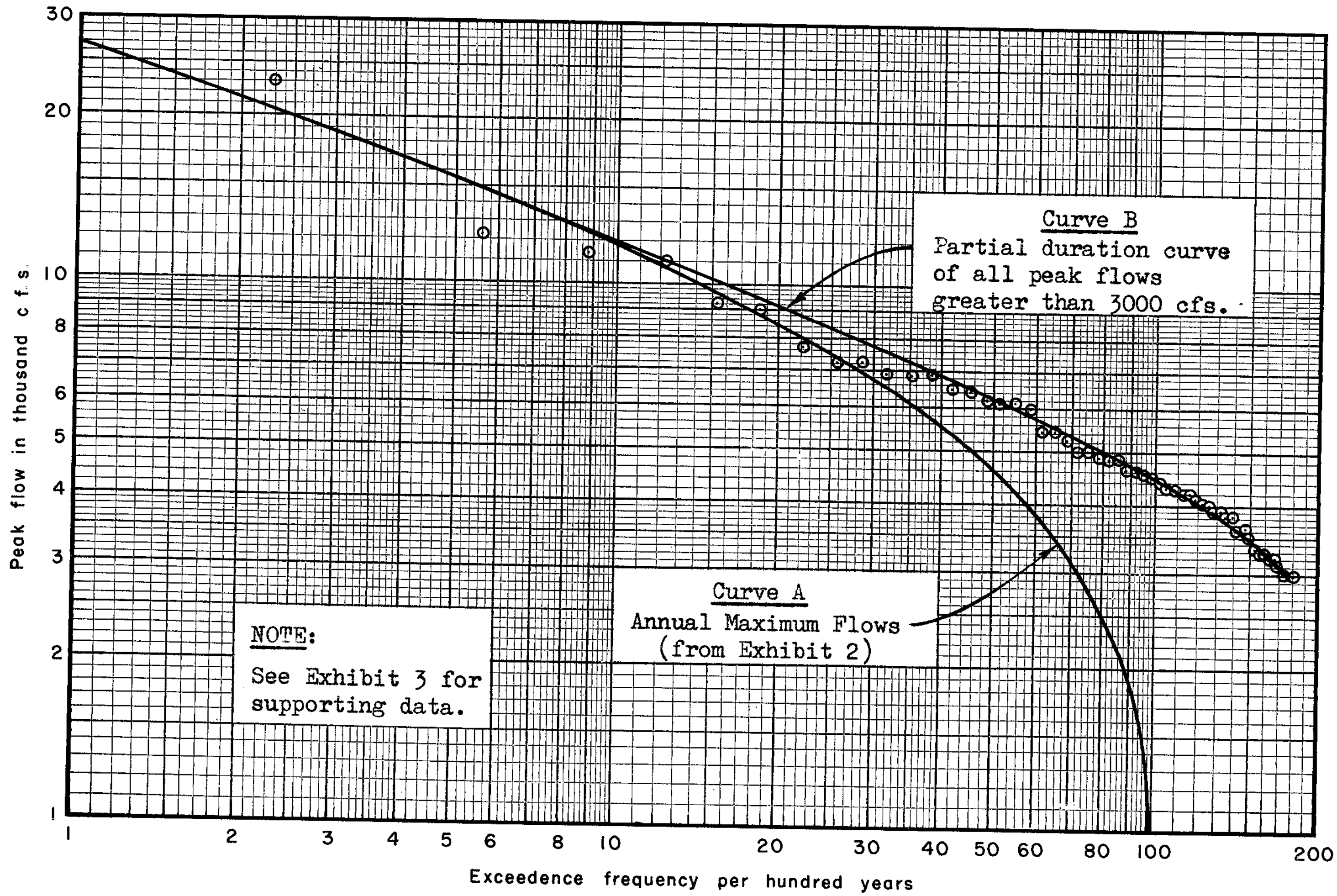
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(9)	P_∞	0.1	1.0	10	50	90	99	99.9
(10)	k (Ex 39)	3.09	2.33	1.28	0	-1.28	-2.33	-3.09
(11)	Log Q (Eq 5)	4.524	4.310	4.014	3.653	3.293	2.996	2.781
(12)	Q, cfs	33,400	20,400	10,300	4,500	1,960	991	604
(13)	P_N, (Ex 40) (Plot Q vs. P_N)	0.20	1.34	10.6	50	89.4	98.67	99.80
(14)	.05 error is S units (Ex 6)	.79	.62	.43	.27	.31	.44	.54
(15)	.05 error, log	.223	.175	.121	.076	.088	.124	.152
(16)	.05 limit-curve value, log	4.744	4.485	4.135	3.729	3.381	3.120	2.933
(17)	.05 limit-curve value, cfs (Plot vs. P_∞)	55,800	30,500	13,600	5,360	2,400	1,320	858
(18)	.95 error in S units (Ex 6)	-.54	-.44	-.31	-.27	-.43	-.62	-.79
(19)	.95 error, log	-.152	-.124	-.088	-.076	-.121	-.175	-.223
(20)	.95 limit-curve value, log	4.372	4.186	3.926	3.577	3.172	2.861	2.558
(21)	.95 limit-curve value, cfs (Plot vs. P_∞)	23,500	15,300	8,480	3,780	1,490	662	361

NOTE: See Exhibit 5 for supporting computations.



ILLUSTRATIVE EXAMPLE

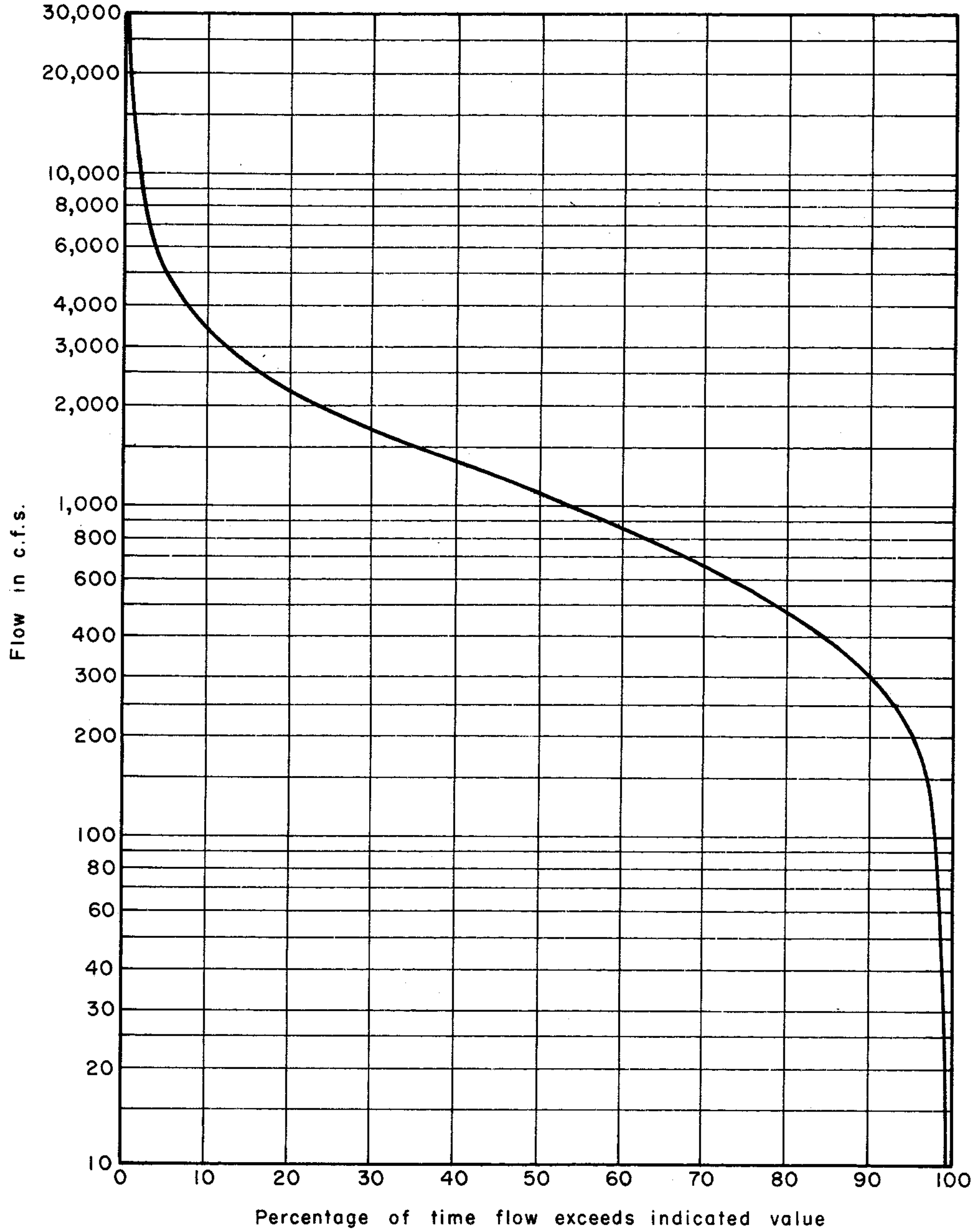
ERROR-LIMIT CURVES



ILLUSTRATIVE EXAMPLE

PARTIAL-DURATION FREQUENCY CURVE

(See par. 4-04)

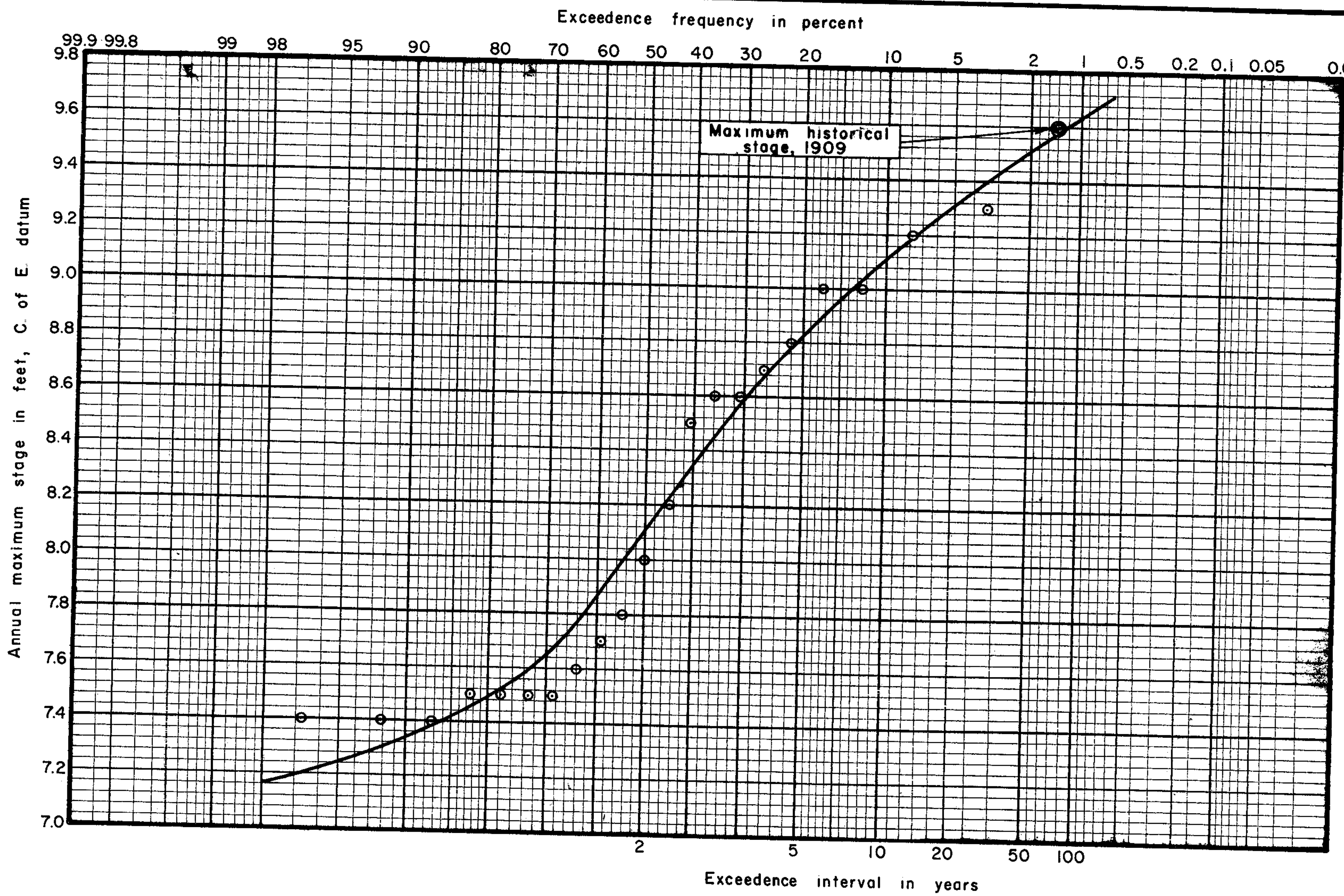


ILLUSTRATIVE EXAMPLE

FLOW DURATION CURVE

RAPPAHANNOCK RIVER AT FREDERICKSBURG, VIRGINIA

(See par. 2-04e)



ILLUSTRATIVE EXAMPLE

RIVER STAGE FREQUENCY CURVE, 1935-1956

(See par. 3-07c)

ILLUSTRATIVE EXAMPLE

ANALYTICAL FREQUENCY COMPUTATION
USING PRE-RECORD DATA

Location: Willamette R at Albany, Oregon
Period of record: 1893-1958 (65 years)
Years of historical flood estimates: 1861, 1887, 1890 (3 largest known)
Period covered by history and record: 1858-1958)(100 years)

(See par. 4-05)

Event No.	k + 4.00 N=100	k + 4.00 N=65	k + 4.00 Smaller	log Q	Event No.	k + 4.00 N=100	k + 4.00 N=65	k + 4.00 Smaller	log Q
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	6.58		6.58	2.53	40		3.84	3.84	1.98
2	6.17		6.17	2.46	41		3.81	3.81	1.97
3	5.96		5.96	2.42	42		3.77	3.77	1.97
4	5.81	6.43	5.81	2.36	43		3.73	3.73	1.96
5	5.70	6.00	5.70	2.36	44		3.69	3.69	1.96
6	5.60	5.77	5.60	2.36	45		3.65	3.65	1.96
7	5.51	5.61	5.51	2.33	46		3.60	3.60	1.94
8	5.44	5.48	5.44	2.32	47		3.56	3.56	1.94
9	5.37	5.38	5.37	2.31	48		3.52	3.52	1.91
10	5.31	5.29	5.29	2.31	49		3.47	3.47	1.91
11	5.25	5.20	5.20	2.29	50		3.43	3.43	1.90
12	5.20	5.13	5.13	2.29	51		3.38	3.38	1.89
13	5.15	5.05	5.05	2.28	52		3.34	3.34	1.88
14	5.10	4.99	4.99	2.26	53		3.29	3.29	1.88
15	5.06	4.93	4.93	2.25	54		3.24	3.24	1.88
16		4.87	4.87	2.23	55		3.18	3.18	1.88
17		4.82	4.82	2.23	56		3.13	3.13	1.86
18		4.76	4.76	2.22	57		3.07	3.07	1.85
19		4.71	4.71	2.22	58		3.01	3.01	1.85
20		4.66	4.66	2.18	59		2.95	2.95	1.84
21		4.62	4.62	2.16	60		2.87	2.87	1.78
22		4.57	4.57	2.14	61		2.80	2.80	1.77
23		4.53	4.53	2.14	62		2.71	2.71	1.76
24		4.48	4.48	2.14	63		2.62	2.62	1.73
25		4.44	4.44	2.13	64		2.52	2.52	1.72
26		4.40	4.40	2.13	65		2.39	2.39	1.72
27		4.35	4.35	2.11	66		2.23	2.23	1.69
28		4.31	4.31	2.11	67		2.00	2.00	1.67
29		4.27	4.27	2.11	68		1.57	1.57	1.61
30		4.23	4.23	2.10					
31		4.19	4.19	2.10	N"			68	68
32		4.16	4.16	2.10	ΣX			277.47	139.73
33		4.12	4.12	2.09	M'			4.080	2.055
34		4.08	4.08	2.08					
35		4.04	4.04	2.07	ΣX^2			1206.2039	290.1517
36		4.00	4.00	2.06	$(\Sigma X)^2/N''$			1132.2000	287.1246
37		3.96	3.96	2.06	diff.			74.0039	3.0271
38		3.92	3.92	2.04					
39		3.88	3.88	1.99					

$S = b = (3.0271/74.0039)^{1/2} = .202$ (Eq. 36)
$M = a = 2.055 - .202(4.080-4.000) = 2.039$ (Eq. 21)

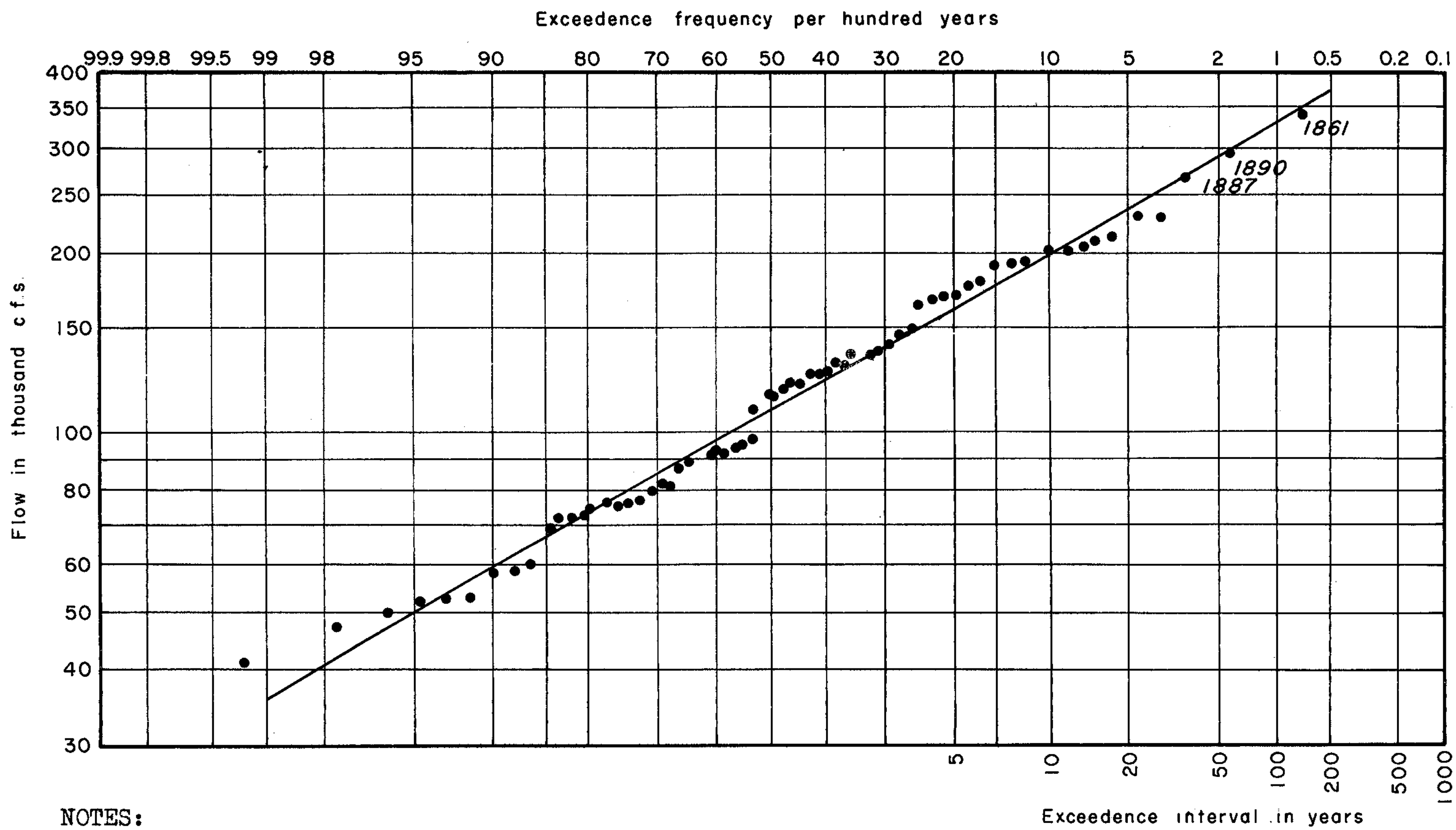
Computation of Frequency Curve (N = 65)

(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
P_N	0.25	1.0	10	50	90	99	99.75
k (Ex. 38)	2.94	2.41	1.31	0.00	-1.31	-2.41	-2.94
Log Q (Eq. 5)	2.633	2.526	2.304	2.039	1.774	1.552	1.445
Q, thous cfs	430	336	201	109	59.4	35.6	27.9

NOTES:

Events numbered 4 to 9 could not have occupied higher positions if the entire 100-year record were available, (but might occupy lower positions). Accordingly, k values were selected as shown above.

4.00 was added to all k values in order that all numbers are positive and have 3 digits for simplicity of machine operation.



NOTES:

1. Curve shown hereon is for illustrative purposes only and was computed from data recorded from 1893 to 1958 and on estimates of three pre-record peak flows. Flows that were partly regulated (since 1941) were adjusted to natural conditions.

2. See Exhibit 11 for computations.

ILLUSTRATIVE EXAMPLE

ANALYTICAL FREQUENCY CURVE USING PRE-RECORD DATA
WILLAMETTE RIVER AT ALBANY, OREGON

(See par. 4-05)

(See par. 4-07)

ILLUSTRATIVE EXAMPLE

ANALYTICAL FREQUENCY COMPUTATION OMITTING LOWER FLOWS

Chronological order			Order of magnitude				
Water Year	Date	Peak flow (cfs)	No.	Plotting position (%)	Peak flow (cfs)	Log of peak	k
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1921	16 May	1250	1	1.9	1890	3.28	2.20
1922	6 May	1380	2	4.7	1780	3.25	1.74
1923	10 May	1450	3	7.4	1480	3.17	1.48
1924	4 May	618	4	10.2	1450	3.16	1.30
1925	8 May	523	5	12.9	1430	3.16	1.15
1926	21 Apr	508	6	15.7	1380	3.14	1.03
1927	30 Apr	1220	7	18.4	1300	3.11	0.91
1928	28 Apr	1180	8	21.1	1280	3.11	0.81
1929	14 May	1060	9	23.9	1250	3.10	0.72
1930	25 Apr	412	10	26.6	1220	3.09	0.63
1931	5 May	170	11	29.4	1180	3.07	0.55
1932	14 May	1480	12	32.1	1090	3.04	0.47
1933	21 May	876	13	34.9	1060	3.02	0.39
1934	21 Jul	113	14	37.6	1020	3.01	0.32
1935	10 May	516	15	40.4	1000	3.00	0.25
1936	4 May	1780	16	43.1	995	3.00	0.17
1937	8 May	1090	17	45.9	985	2.99	0.11
1938	22 Apr	760	18	48.6	876	2.94	0.04
1939	30 Apr	397	19	51.3	788		
1940	21 Apr	282	20	54.1	788		
1941	2 May	353	21	56.9	760		
1942	13 Apr	597	22	59.6	678		
1943	23 Apr	995	23	62.3	618		
1944	14 May	611	24	65.1	611		
1945	4 May	985	25	67.9	611		
1946	18 Apr	1430	26	70.6	597		
1947	3 May	788	27	73.4	523		
1948	17 May	1280	28	76.1	516		
1949	24 Apr	1020	29	78.9	508		
1950	18 May	1300	30	81.6	412		
1951	12 May	1000	31	84.3	409		
1952	3 May	1890	32	87.1	397		
1953	29 May	611	33	89.8	353		
1954	25 Apr	409	34	92.6	282		
1955	6 May	788	35	95.3	170		
1956	24 Dec	678	36	98.1	113		

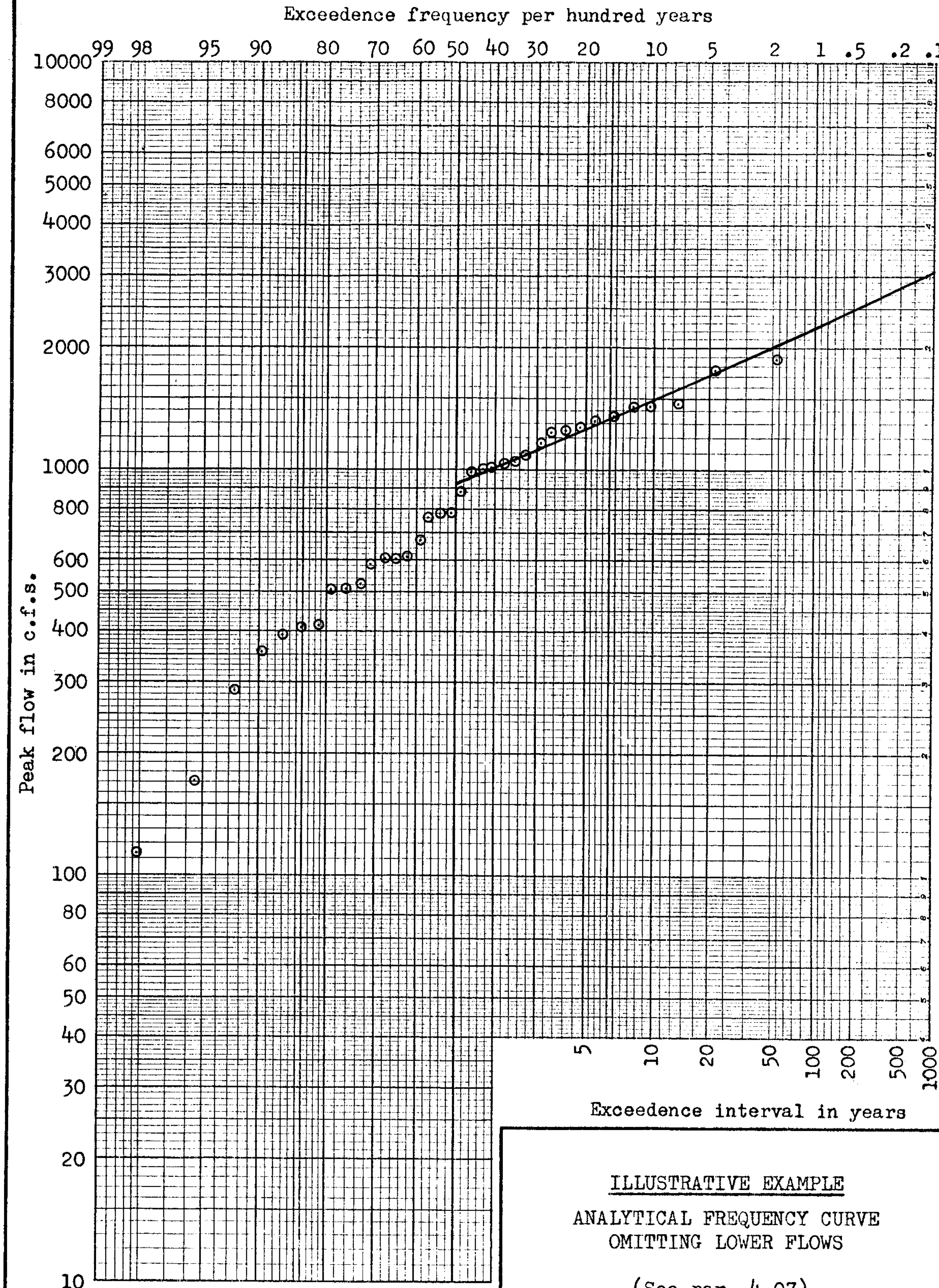
	(7)	(8)
N"	18	18
ΣX	55.64	14.27
M"	3.091	0.793
ΣX^2	172.1336	17.4139
$(\Sigma X)^2/N''$	171.9894	11.3129
Σx^2	.1442	6.1010

$S = b = (.144/6.101)^{1/2} = 0.154$

$M = a = 3.091 - .154(.793) = 2.969$

Computation of curve (N=36)

(9)	(10)	(11)	(12)
P_N	k	log Q	Q
.0025	3.04	3.437	2,740
.01	2.47	3.349	2,230
.1	1.32	3.172	1,490
.5	0	2.969	931



NOTE: See Exhibit 13 for computations.

ILLUSTRATIVE EXAMPLE

ANALYTICAL FREQUENCY CURVE
OMITTING LOWER FLOWS

(See par. 4-07)

MAXIMUM-RUNOFF VOLUME FREQUENCY DATA--CHRONOLOGICAL ORDER

Average Flows in c.f.s. at: Mill Creek nr. Los Molinos, California

Drainage area: 134 sq. mi. Runoff type: All-season Basin regulation: Natural

Water Year (1)	Peak date (2)	Peak flow (3)	1-day date (4)	1-day flow (5)	3-day first day (6)	3-day flow (7)	10-day first day (8)	10-day flow (9)	30-day first day (10)	30-day flow (11)	90-day first day (12)	90-day flow (13)	Water-yr. flow (14)
1928-29	3 Feb.	1520	3 Feb.	965	2 Feb.	719	1 Feb.	377	22 Apr.	292	9 Mar.	225	150
29-30	15 Dec.	6000	15 Dec.	4080	14 Dec.	2400	10 Dec.	1270	10 Dec.	590	10 Dec.	475	255
30-31	23 Jan.	1500	23 Jan.	1200	22 Jan.	650	11 Mar.	315	11 Mar.	235	27 Feb.	186	121
31-32	24 Dec.	5440	27 Dec.	2160	26 Dec.	1380	23 Dec.	1040	21 Dec.	480	18 Mar.	380	214
32-33	16 Mar.	1080	28 Mar.	662	28 Mar.	472	25 May.	386	20 May.	330	27 Mar.	280	150
33-34	29 Dec.	2630	29 Dec.	1730	29 Dec.	1300	29 Dec.	745	27 Mar.	335	7 Feb.	290	166
34-35	4 Jan.	4010	4 Jan.	2300	7 Apr.	1470	7 Apr.	958	3 Apr.	760	28 Feb.	565	290
35-36	21 Feb.	4380	21 Feb.	2660	21 Feb.	1920	14 Feb.	1220	12 Feb.	716	9 Jan.	513	270
36-37	14 Feb.	3310	4 Feb.	1540	4 Feb.	902	12 Mar.	584	1 May	520	11 Mar.	478	219
37-38	11 Dec.	23000	11 Dec.	12300	10 Dec.	7340	10 Dec.	2750	10 Dec.	1100	12 Mar.	890	566
38-39	8 Mar.	1260	3 Dec.	594	8 Mar.	453	19 Mar.	350	8 Mar.	329	7 Mar.	260	156
39-40	28 Feb.	11400	27 Feb.	7640	27 Feb.	6150	26 Feb.	2580	3 Feb.	1200	1 Jan.	915	383
40-41	10 Feb.	12200	11 Feb.	5980	10 Feb.	4730	9 Feb.	1990	8 Feb.	1310	8 Feb.	605	467
41-42	6 Feb.	11000	6 Feb.	5690	5 Feb.	3250	1 Feb.	1810	22 Jan.	1160	30 Nov.	825	443
42-43	8 Mar.	6970	23 Jan.	3770	21 Jan.	3060	21 Jan.	1660	21 Mar.	820	5 Mar.	654	373
43-44	4 Mar.	3220	4 Mar.	1720	3 Mar.	890	29 Feb.	515	29 Apr.	373	29 Feb.	330	198
44-45	5 Feb.	3230	5 Feb.	1580	1 Feb.	1260	1 Feb.	890	31 Jan.	540	1 Feb.	392	170
45-46	21 Dec.	6180	27 Dec.	3100	27 Dec.	2320	21 Dec.	1760	21 Dec.	890	30 Oct.	470	294
46-47	12 Feb.	4070	12 Feb.	2590	11 Feb.	1380	11 Feb.	630	11 Feb.	400	11 Feb.	350	192
47-48	23 Mar.	7320	23 Mar.	3650	23 Mar.	2040	23 Mar.	865	23 Mar.	770	23 Mar.	682	310
48-49	11 Mar.	3870	23 Mar.	1810	10 Mar.	1300	10 Mar.	700	2 Mar.	410	2 Mar.	407	195
49-50	4 Feb.	4430	4 Feb.	3210	4 Feb.	2360	4 Feb.	990	17 Jan.	590	18 Mar.	452	251
50-51	16 Nov.	3870	22 Jan.	2120	19 Nov.	1540	3 Dec.	1060	16 Nov.	840	1 Dec.	565	343
51-52	26 Dec.	5280	1 Dec.	3040	1 Feb.	2330	31 Jan.	1250	24 Jan.	860	14 Mar.	726	445
52-53	9 Jan.	7710	9 Jan.	5240	8 Jan.	3200	7 Jan.	1900	26 Dec.	1080	1 Dec.	570	348
53-54	17 Jan.	4910	5 Apr.	2290	4 Apr.	1870	3 Apr.	1000	9 Apr.	740	12 Feb.	633	318
54-55	11 Nov.	2480	15 Nov.	1060	14 Nov.	565	2 Dec.	480	2 May	402	25 Mar.	325	202
55-56	22 Dec.	9180	22 Dec.	6770	21 Dec.	5060	18 Dec.	3260	19 Dec.	1880	18 Dec.	1080	493
56-57	24 Feb.	6140	24 Feb.	3840	24 Feb.	2630	24 Feb.	1340	23 Feb.	806	23 Feb.	528	257
57-58	24 Feb.	6880	24 Feb.	3580	24 Feb.	2530	16 Feb.	1560	29 Jan.	1290	24 Jan.	918	488

Comp. by: R.P.L. Date: May 1959 Sheet: Station: USGS 11-3810

SPK FORM 237 6 MAY 59

EXHIBIT 15

Comp. by: R.P.L. Date: May 1959 Sheet: Station: USGS 11-3810

MAXIMUM-RUNOFF VOLUME FREQUENCY DATA--ORDER OF MAGNITUDE

Average Flows in c.f.s. at: Mill Creek nr Los Molinos, Calif.

Annual-event curve data								Partial-duration curve data				
								Additional flows				
plotting position (1)	peak flow (2)	1-day flow (3)	3-day flow (4)	10-day flow (5)	30-day flow (6)	90-day flow (7)	water-year flow (8)	water-yr. (9)	date (10)	flow (11)	plotting position (12)	flow (13)
2.3	23,000	12,300	7,340	3,260	1,880	1,080	566	1934-35	28 Feb	3,190	2.3	23,000
5.6	12,200	7,640	6,150	2,750	1,310	918	493	1934-35	8 Apr	3,040	5.6	12,200
8.9	11,400	6,770	5,060	2,580	1,290	915	488	1935-36	11 Jan	3,930	8.9	11,400
12.2	11,000	5,980	4,730	1,990	1,200	890	467	1937-38	20 Nov	4,700	12.2	11,000
15.4	9,180	5,690	3,250	1,990	1,160	825	445	1937-38	2 Feb	5,050	15.4	9,360
18.7	7,710	5,240	3,200	1,810	1,100	726	443	1937-38	23 Mar	4,950	18.7	9,180
22.0	7,320	4,080	3,060	1,760	1,080	682	383	1939-40	2 Jan	4,600	22.0	7,710
25.3	6,970	3,840	2,630	1,660	890	654	373	1939-40	30 Mar	9,360	25.3	7,320
28.6	6,880	3,770	2,530	1,560	860	633	348	1940-41	24 Dec	6,240	28.6	7,260
31.9	6,180	3,650	2,400	1,340	840	605	343	1940-41	1 Mar	4,250	31.9	6,970
35.2	6,140	3,580	2,360	1,270	820	570	318	1940-41	4 Apr	7,260	35.2	6,910
38.5	6,000	3,210	2,330	1,250	806	565	310	1941-42	3 Dec	4,130	38.5	6,880
41.8	5,440	3,100	2,320	1,220	770	565	294	1941-42	16 Dec	6,910	41.8	6,480
45.1	5,280	3,040	2,040	1,060	760	528	290	1941-42	27 Jan	5,450	45.1	6,450
48.4	4,910	2,660	1,920	1,040	740	513	270	1942-43	21 Jan	6,450	48.4	6,240
51.6	4,430	2,590	1,870	1,000	716	478	257	1945-46	4 Dec	3,660	51.7	6,180-
54.9	4,380	2,300	1,540	990	590	475	255	1947-48	28 Apr	3,380	55.0	6,140
58.2	4,070	2,290	1,470	958	590	470	251	1950-51	22 Jan	3,510	58.3	6,000
61.5	4,010	2,160	1,380	890	540	452	219	1950-51	11 Feb	3,660	61.7	5,450
64.8	3,870	2,120	1,380	865	520	407	214	1951-52	1 Dec	4,930	65.0	5,440
68.1	3,870	1,810	1,300	745	480	392	202	1951-52	1 Feb	4,650	68.3	5,280
71.4	3,310	1,730	1,300	700	410	380	198	1952-53	27 Apr	3,070	71.7	5,050
74.7	3,230	1,720	1,260	630	402	350	195	1953-54	13 Feb	3,300	75.0	5,020
78.0	3,220	1,580	902	584	400	330	192	1953-54	4 Apr	4,240	78.3	4,950
81.3	2,630	1,540	890	515	373	325	170	1955-56	7 Jan	5,020	81.7	4,930
84.6	2,480	1,200	719	480	335	290	166	1955-56	22 Feb	6,480	85.0	4,910
87.8	1,520	1,060	650	386	330	280	156	1957-58	26 Jan	3,060	88.3	4,700
91.1	1,500	965	565	377	329	260	150	1957-58	12 Feb	4,330	91.7	4,650
94.4	1,260	662	472	350	292	225	150	1957-58	21 Mar	4,540	95.0	4,600
97.7	1,080	594	453	315	235	186	121	1957-58	2 Apr	3,970	98.3	4,540

SPK FORM 6 MAY 59 237a

EXHIBIT 16

Comp. by: RPL   Date: May 1959   Sheet: Station: USGS 11-3810

MAXIMUM-RUNOFF VOLUME FREQUENCY DATA - LOGARITHMS

Logarithms of Average Flow in c.f.s.

Station 1: Mill Creek nr. Los Molinos, Calif.   Station 2 (base sta.): Feather River at Bidwell Bar, Calif.

Water year (1)	Peak Sta. 1 (2)	Peak Sta. 2 (3)	1-day Sta. 1 (4)	1-day Sta. 2 (5)	3-day Sta. 1 (6)	3-day Sta. 2 (7)	10-day Sta. 1 (8)	10-day Sta. 2 (9)	30-day Sta. 1 (10)	30-day Sta. 2 (11)	90-day Sta. 1 (12)	90-day Sta. 2 (13)	Water-year Sta. 1 (14)	Water-year Sta. 2 (15)
1928-29	3.18		2.98	3.67	2.86	3.50	2.58	3.45	2.47	3.39	2.35	3.26	2.18	2.87
29-30	3.78		3.61	4.33	3.38	4.20	3.10	4.00	2.77	3.68	2.68	3.60	2.41	3.27
30-31	3.18		3.08	3.61	2.81	3.56	2.50	3.38	2.37	3.23	2.27	3.07	2.08	2.72
31-32	3.74		3.33	3.94	3.14	3.89	3.02	3.76	2.68	3.67	2.58	3.64	2.33	3.22
32-33	3.03		2.82	3.64	2.67	3.63	2.59	3.56	2.52	3.44	2.45	3.34	2.18	2.90
33-34	3.42		3.24	3.88	3.11	3.76	2.87	3.49	2.53	3.36	2.46	3.28	2.22	2.92
34-35	3.60		3.36	4.33	3.17	4.21	2.98	4.06	2.88	3.96	2.75	3.75	2.46	3.29
35-36	3.64		3.42	4.33	3.28	4.26	3.09	4.04	2.86	3.67	2.71	3.71	2.43	3.30
36-37	3.52		3.19	3.89	2.96	3.83	2.77	3.74	2.72	3.68	2.68	3.60	2.34	3.14
37-38	4.36		4.09	4.86	3.87	4.64	3.44	4.23	3.04	4.05	2.95	3.99	2.75	3.61
38-39	3.10		2.77	3.48	2.66	3.47	2.54	3.45	2.52	3.41	2.41	3.22	2.19	2.84
39-40	4.06		3.88	4.72	3.79	4.57	3.41	4.33	3.08	3.98	2.96	3.89	2.58	3.42
40-41	4.09		3.78	4.36	3.68	4.26	3.30	4.02	3.12	3.90	2.78	3.77	2.67	3.45
41-42	4.04		3.76	4.44	3.51	4.30	3.26	4.03	3.06	3.96	2.92	3.75	2.65	3.47
42-43	3.84		3.58	4.43	3.49	4.40	3.22	4.17	2.91	3.89	2.82	3.82	2.57	3.42
43-44	3.51		3.24	3.77	2.95	3.73	2.71	3.69	2.57	3.57	2.52	3.45	2.30	3.04
44-45	3.51		3.20	4.33	3.10	4.19	2.95	3.97	2.73	3.72	2.59	3.57	2.23	3.24
45-46	3.79		3.49	4.28	3.37	4.21	3.25	4.10	2.95	3.83	2.67	3.54	2.47	3.28
46-47	3.61		3.41	4.08	3.14	3.96	2.80	3.66	2.60	3.55	2.54	3.43	2.28	3.02
47-48	3.86		3.56	4.18	3.31	4.05	2.94	3.93	2.89	3.81	2.83	3.66	2.49	3.22
48-49	3.59		3.26	3.77	3.11	3.76	2.84	3.74	2.61	3.66	2.61	3.52	2.29	3.05
49-50	3.65		3.51	4.18	3.37	4.08	3.00	3.84	2.77	3.74	2.66	3.64	2.40	3.26
50-51	3.59		3.33	4.51	3.19	4.40	3.02	4.12	2.92	4.02	2.75	3.81	2.54	3.44
51-52	3.72		3.48	4.23	3.37	4.20	3.10	4.15	2.93	4.11	2.86	3.97	2.65	3.58
52-53	3.89		3.72	4.52	3.50	4.29	3.28	4.08	3.03	3.87	2.76	3.61	2.54	3.35
53-54	3.69		3.36	4.29	3.27	4.19	3.00	3.91	2.87	3.77	2.80	3.67	2.50	3.23
54-55	3.39		3.03	3.72	2.75	3.70	2.68	3.64	2.60	3.51	2.51	3.36	2.31	2.99
55-56	3.96		3.83	4.91	3.70	4.78	3.51	4.52	3.27	4.23	3.03	3.95	2.69	3.57
56-57	3.79		3.58	4.37	3.42	4.33	3.13	4.08	2.91	3.90	2.72	3.67	2.41	3.20
57-58	3.84		3.55	4.43	3.40	4.30	3.19	4.13	3.11	3.98	2.96	3.88	2.69	3.50

SPK FORM 6 MAY 59 237b

EXHIBIT 17

Period of record: 1928-58   Base station period: 1911-58

By: R.P.L.   Date: May 1959   Station No. USGS 11-3810

MAXIMUM-RUNOFF VOLUME FREQUENCY COMPUTATIONS

Station I: Mill Creek nr Los Molinos, Calif.   Station 2 (base sta.): Feather River at Bidwell Bar, Calif.

(32) $R^2=(\Sigma x_1 x_2)^2/\Sigma x_1^2 \Sigma x_2^2$ $(1-\bar{R}^2)=(1-R^2)(N-1)/(N-2)$ $S_1'-S_1=(S_2'-S_2)R^2(S_1/S_2)$ $M_1'-M_1=(M_2'-M_2)R(S_1/S_2)$ $N_1'=N_1/[1-(N_2'-N_1)\bar{R}^2/N_2']$

		Peak		1-day		3-day		10-day		30-day		90-day		Water-year	
		Sta. 1	Sta. 2	Sta. 1	Sta. 2	Sta. 1	Sta. 2	Sta. 1	Sta. 2	Sta. 1	Sta. 2	Sta. 1	Sta. 2	Sta. 1	Sta. 2
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
(33)	N	30	30	30	30	30	30	30	30	30	30	30	30	30	30
(34)	ΣX	109.97	Use 1-day values at Sta. 1 as base (assume $\bar{R}^2$ = 1)	102.44	125.48	97.33	122.65	90.07	117.27	84.29	112.54	80.58	108.42	72.83	96.81
(35)	M	3.666		3.415	4.183	3.244	4.088	3.002	3.909	2.810	3.751	2.686	3.614	2.428	3.227
(36)	ΣX^2	405.781		352.561	528.884	318.688	504.802	272.605	460.721	238.321	423.907	217.513	393.424	177.758	314.038
(37)	$(\Sigma X)^2/N$	403.113		349.798	524.841	315.771	501.434	270.420	458.408	236.827	422.175	216.438	391.830	176.807	312.406
(38)	Σx^2	2.668		2.763	4.043	2.917	3.368	2.185	2.313	1.494	1.732	1.075	1.594	.951	1.632
(39)	$\Sigma x^2/N-1$	.0920		.0953	.1394	.1006	.1161	.0753	.0798	.0515	.0597	.0371	.0550	.0328	.0563
(40)	S	.303		.309	.373	.317	.341	.274	.283	.227	.244	.193	.234	.181	.237
(41)	$\Sigma X_1 X_2$			431.454		400.758		354.161		317.701		292.456		236.206	
(42)	$\Sigma X_1 \Sigma X_2/N$			428.472		397.918		352.084		316.200		291.216		235.022	
(43)	$\Sigma x_1 x_2$			2.982		2.840		2.077		1.501		1.240		1.184	
(44)	R^2			.796		.821		.854		.871		.897		.903	
(45)	$\bar{R}^2$	1.00		.789		.815		.849		.866		.893		.900	
(46)	$S'-S$	-.022	-.022	-.022	-.034	-.020	-.027	-.015	-.019	-.008	-.010	-.002	-.003	-.003	-.004
(47)	S'	.281		.287	.339	.297	.314	.259	.264	.219	.234	.191	.231	.178	.233
(48)	S'(Adj.)	.288		.288		.288		.254		.227		.210		.172	
(49)	$M'-M$	-.005	-.005	-.005	-.007	-.001	-.002	0	0	.003	+.004	-.001	-.001	-.005	-.007
(50)	M'	3.661	3.410	3.410	4.176	3.243	4.086	3.002	3.909	2.813	3.755	2.685	3.613	2.423	3.220
(51)	N'	42	42	42	47	42.5	47	43.3	47	43.7	47	44.3	47	44.5	47
(52)	g	.00		-.04		-.12		-.23		-.32		-.37		-.40	

	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)
(53)	P_∞	$P_{N'}$	Log Q	Q	Log Q	Q	Log Q	Q	Log Q	Q	Log Q	Q	Log Q	Q	Log Q	Q
(54)	.01	.032	4.735	54,300	4.461	28,900	4.245	17,600	3.830	6,760	3.512	3,250	3.311	2,050	2.925	841
(55)	.1	.19	4.551	35,600	4.283	19,200	4.084	12,100	3.706	5,080	3.421	2,640	3.227	1,690	2.860	724
(56)	1.0	1.31	4.332	21,500	4.072	11,800	3.888	7,730	3.551	3,560	3.287	1,940	3.116	1,310	2.772	592
(57)	10.0	10.6	4.030	10,700	3.776	5,970	3.606	4,040	3.320	2,090	3.092	1,240	2.941	873	2.633	430
(58)	50.0	50.0	3.661	4,580	3.413	2,590	3.249	1,770	3.010	1,020	2.824	667	2.698	499	2.433	271
(59)	90.0	89.4	3.292	1,960	3.041	1,100	2.871	743	2.672	470	2.516	328	2.408	256	2.196	157
(60)	99.0	98.69	2.990	977	2.730	537	2.546	352	2.367	233	2.232	171	2.139	138	1.972	94
(61)	99.9	99.81	2.771	590	2.503	318	2.304	201	2.133	136	2.007	102	1.923	84	1.792	62
(62)	99.99	99.968	2.587	386	2.310	204	2.094	124	1.928	85	1.810	65	1.734	54	1.632	43

SPK FORM 5 FEB 60 237c   (Replaces SPK Form 237c dtd 6 May 59)

COMPUTATION PROCEDURE

1. __Annual-event Data.__ For each water year or seasonal portion of a water year at a given site, tabulate to three significant figures the maximum unregulated runoff volume in average-flow units for each duration with date as indicated on Exhibit 15. Long-duration volumes need not cover the time period of the corresponding short-duration volumes. It is usually best to restrict the tabulation to the same years for all durations.

2. __Partial-duration Data.__ If a partial-duration curve of peak or daily flows is also desired, tabulate all additional peak (or daily) flows exceeding a selected non-damage base value and separated from larger flows by a minimum specified time period, as indicated on Exhibit 16. Unless doing so would require discarding damaging flows, the base should be selected or adjusted to provide a total of N (number of years) events, including annual-event flows exceeding that base. A value exceeded by two-thirds of the annual event flows is almost always low enough for a first approximation of the base.

3. __Plotting of Points.__ Arrange each set of annual-event flows and the set of partial-duration flows (including the corresponding annual-event flows) in the order of magnitude and tabulate plotting positions from Exhibit 37 as shown on Exhibit 16. Plot all the above data on logarithmic probability grid as shown on Exhibit 20.

4. __Logarithms.__ Tabulate common logarithms to two decimal places for all tabulated annual-event flows at the site (Station 1) and for corresponding annual maximum flows at a selected long-record base station (Station 2) as shown on Exhibit 17.

5. __Sums, Squares and Cross Products.__ Logarithms are designated X_1 for Station 1 and X_2 for Station 2. Compute the sum (ΣX, line 34) and the sum of squares (ΣX^2, line 36) of logarithms in each column. Compute the sum of cross products of Station 1 and Station 2 logarithms for each duration ($\Sigma X_1 X_2$, line 41). These are all computed in a single operation as described in paragraph 9-02b. Tabulate number of items (N, line 33) of data used.

6. __Mean (M) and Standard Deviation (S).__ For each column, compute the mean (M, line 35) from $M = \Sigma X/N$ and the standard deviation (S, line 40) from $S = \left[\frac{\Sigma X^2 - (\Sigma X)^2/N}{N-1}\right]^{\frac{1}{2}}$. To compute S: square ΣX, divide by N, and enter on line 37. Subtract this value from ΣX^2 to obtain Σx^2 and enter on line 38. Divide Σx^2 by (N-1) and enter on line 39. Take the square root to obtain S and enter on line 40.

7. __Determination Coefficient ($\bar{R}^2$).__ For each duration, compute the determination coefficient between corresponding site and base-station logarithms by use of the first two equations on line 32 as follows: Multiply ΣX_1 by ΣX_2, line 34, divide by N and enter on line 42. Subtract this from $\Sigma X_1 X_2$ to obtain $\Sigma x_1 x_2$ and enter on line 43. Square $\Sigma x_1 x_2$ and divide by the product of Σx_1^2 and Σx_2^2, line 38, to obtain R^2 and enter on line 44. Subtract R^2 from 1.0 and multiply by the ratio (N-1)/(N-2). Subtract the result from 1.0 to obtain $\bar{R}^2$ and enter on line 45.

8. __Extended Statistics, M', S', and N'.__ For each duration, compute the long-period statistics by use of the last three equations in line 32 as follows. In the Station 2 column, enter the long-period mean, M_2', on line 50, the long-period standard deviation, S_2', on line 47, and the long-period number of events, N_2', on line 51.
To compute S_1': subtract S_2 from S_2' and enter on line 46. Multiply by R^2 and by the ratio S_1/S_2, line 40, and enter the result, $S_1' - S_1$, in the Station 1 column on line 46. Add algebraically to S_1 to obtain S_1' and enter on line 47.
To compute M_1': subtract M_2 from M_2' and enter on line 49. Multiply this difference by R and by the ratio S_1'/S_2', line 47, and enter the result, $M_1' - M_1$, on line 49. Add algebraically to M_1 and enter result, M_1', on line 50.
To compute N_1': Multiply the difference between N_2' and N_1 by $\bar{R}^2$ and divide by N_2. Subtract results from 1.0 and divide N_1 by this value, and enter on line 51.

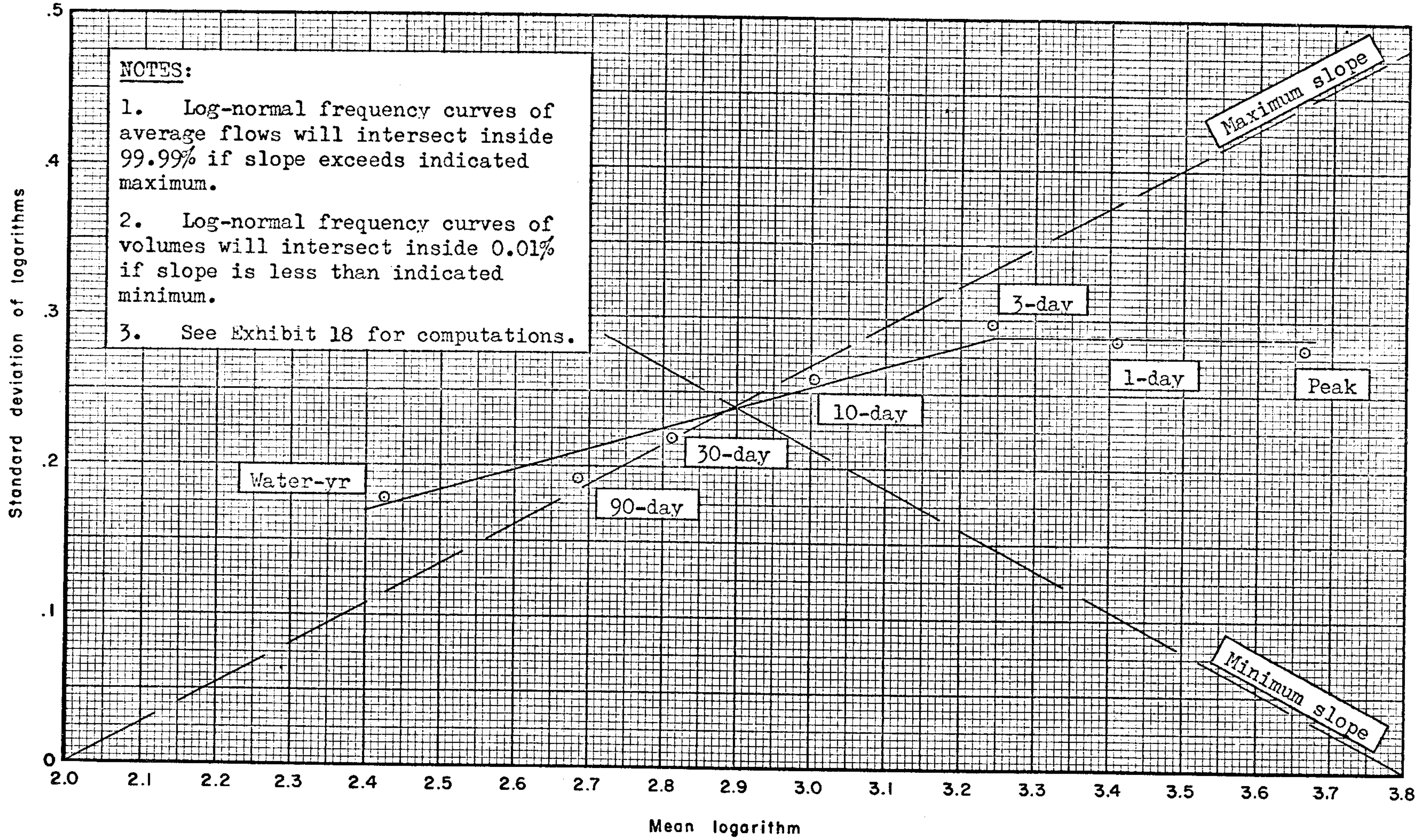
9. __Adjusted Standard Deviation (S' Adj).__ Plot the extended standard deviation S_1' against the extended mean M_1' for each duration as illustrated on Exhibit 19 and draw a straight or broken line to smooth the S' values. This will assure consistency among frequency curves for the various durations. Do not exceed the maximum indicated slope. Tabulate adjusted S_1' values on line 48.

10. __Skew Coefficients (g).__ Select a skew coefficient for each duration from paragraph 6-03 or from a regional study as described in paragraph 7-11. Tabulate in the station 1 columns on line 52.

11. __Computation of Curves.__ For each duration, compute the logarithm of flow for selected values of P_∞ by the equation, $\log Q = M + kS$. The values of P_∞ shown in column 16 are ordinarily satisfactory. Obtain corresponding values of k from Exhibit 39 for each value of g. Multiply each k value in turn by S_1' (adj), line 48, and add to M_1' to obtain log Q. Tabulate in log-Q column. Tabulate corresponding anti-logarithms (Q) in the column headed Q. For each value of P_∞, tabulate the corresponding value of P_N from Exhibit 40, using the average value of N_1' for all durations.

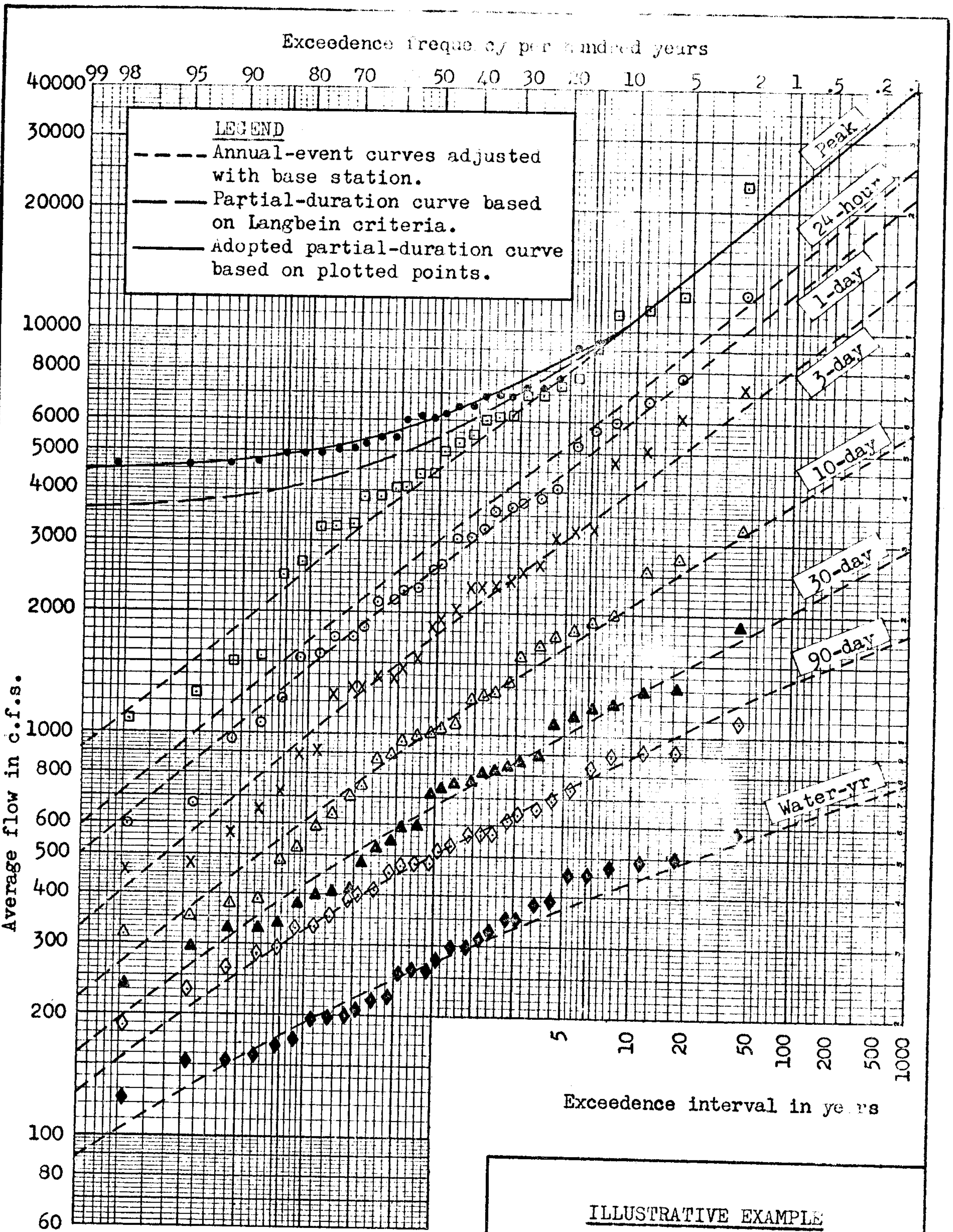
12. __Plotting of Curves.__ Plot flows, Q, against corresponding values of P_N for each duration and draw smooth curves as shown on Exhibit 20. Check curves against points plotted in step 3 for possible arithmetic mistakes. If there is a radical departure of points from the curves, examine the hydrologic features of the basin for possible special treatment as described in paragraph 4-07.

Revised Jan. 1966



STANDARD DEVIATION ADJUSTMENT

(See par. 6-03 e)



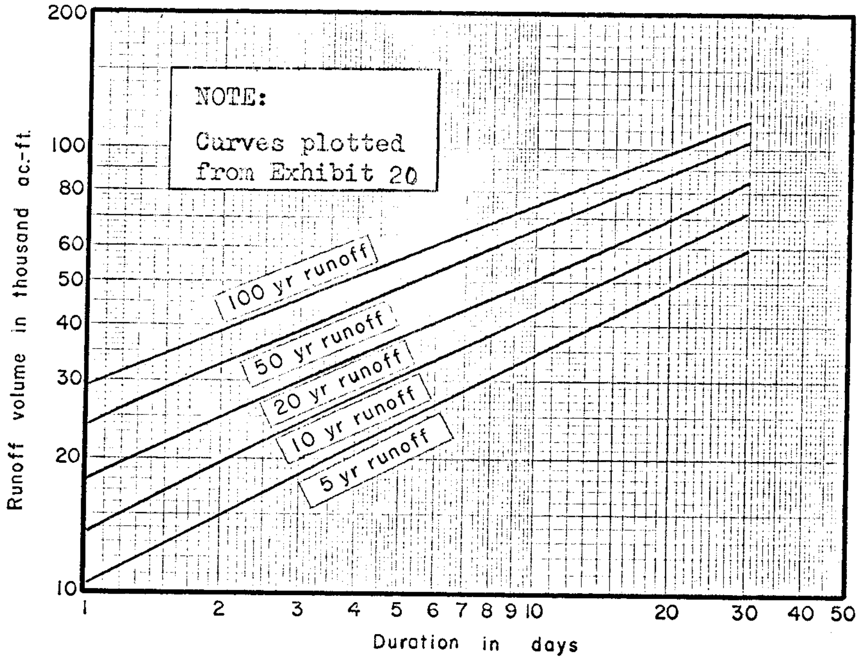
NOTES: 1. Annual-event curves adjusted using record of Feather River at Bidwell Bar.

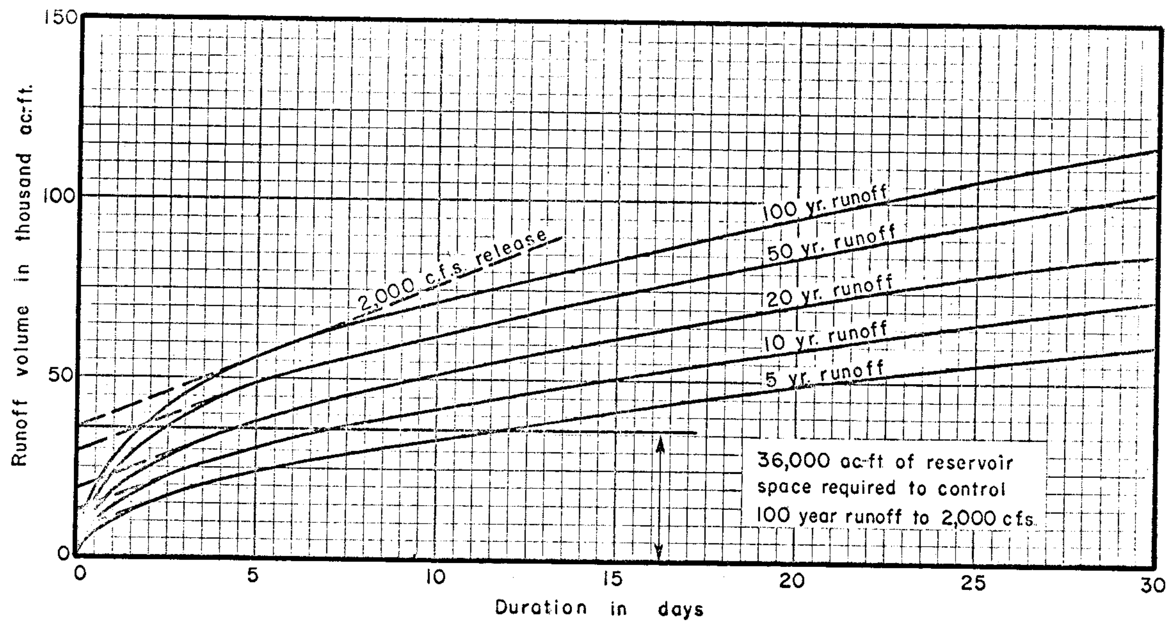
2. See Exhibit 18 for supporting computations.

ILLUSTRATIVE EXAMPLE

MAX-RUNOFF VOLUME FREQUENCY CURVES
MILL CREEK NR LOS MOLINOS

(See par. 6-04)





ILLUSTRATIVE EXAMPLE

VOLUME-DURATION CURVES
MILL CR NEAR LOS MOLINOS, CALIFORNIA

(See par. 6-06)

ILLUSTRATIVE EXAMPLE

REGIONAL FREQUENCY CORRELATION

(See secs. 7 & 9)

$X_1 = 1 + \log.\ S$ $\quad X_2 = \log.\ D.A.$ $\quad X_3 = \log.$ No. rainy days per year

Sta. No. (1)	X_2 (2)	X_3 (3)	X_1 (4)	Sta. No. (5)	X_2 (6)	X_3 (7)	X_1 (8)
1	1.61	2.11	0.29	33	1.94	1.87	0.20
2	2.89	2.12	0.18	34	2.73	1.36	0.58
3	4.38	2.11	0.17	35	3.63	1.81	0.64
4	3.20	2.04	0.44	36	1.91	1.58	0.37
5	3.92	2.07	0.38	37	2.26	1.48	0.27
6	1.61	2.04	0.37	38	2.97	1.89	0.54
7	3.21	2.09	0.30	39	0.70	1.32	0.63
8	3.65	1.99	0.35	40	0.30	1.54	0.78
9	3.23	2.15	0.16	41	3.38	1.62	0.46
10	4.33	2.08	0.11	42	2.87	2.03	0.44
11	1.60	2.09	0.32	43	2.42	2.26	0.24
12	2.82	2.00	0.34	44	4.53	1.93	-0.03
13	2.40	2.00	0.25	45	3.04	1.78	0.30
14	3.69	2.09	0.43	46	4.13	2.00	0.17
15	2.18	2.19	0.27	47	1.49	2.01	0.14
16	2.09	2.17	0.25	48	5.37	1.95	0.10
17	4.48	1.91	0.52	49	1.36	2.11	0.27
18	4.95	1.95	0.18	50	2.31	2.23	0.18
19	2.21	1.97	0.39				
20	3.41	2.08	0.40	ΣX	147.55	96.24	17.89
21	4.82	1.88	0.25	M	2.951	1.925	0.358
22	1.78	1.93	0.23	ΣXX_2	503.7779	285.5627	51.1527
23	4.39	1.74	0.54	$\Sigma X \Sigma X_2/N$	435.4200	284.0042	52.7934
24	3.23	2.01	0.51	Σxx_2	68.3579	1.5585	-1.6407
25	3.58	2.04	0.45				
26	1.64	1.78	0.63	ΣXX_3		187.5912	33.2598
27	4.58	1.76	0.45	$\Sigma X \Sigma X_3/N$		185.2428	34.4347
28	3.26	1.93	0.59	Σxx_3	1.5585	2.3484	-1.1749
29	4.29	1.81	0.46				
30	1.23	1.89	0.32	ΣXX_1			8.1635
31	3.44	1.48	0.96	$\Sigma X \Sigma X_1/N$			6.4010
32	2.11	1.97	0.12	Σxx_1	-1.6407	-1.1749	1.7625

$$\left.\begin{array}{l} 68.4\ b_2 + 1.56\ b_3 = -1.64 \quad \text{(Eq. 23)} \\ 1.56\ b_2 + 2.35\ b_3 = -1.17 \quad \text{(Eq. 24)} \end{array}\right\} \quad \begin{array}{l} b_2 = -.013 \\ b_3 = -.49 \end{array}$$

$$a = 0.36 + .013\ (2.95) + .49\ (1.92) = 1.34 \quad \text{(Eq. 28)}$$

$$X_1 = 1.34 - .013X_2 - .49X_3 \quad \text{(Regression Equation, eq. 22)}$$

$$R^2 = \frac{-.013\ (-1.64) - .49\ (-1.17)}{1.76} = .338 \quad \text{(Eq. 32)}$$

$$\bar{R}^2 = 1 - (.662)\ 49/47 = .310 \quad \text{(Eq. 31)} \qquad \bar{R} = .56$$

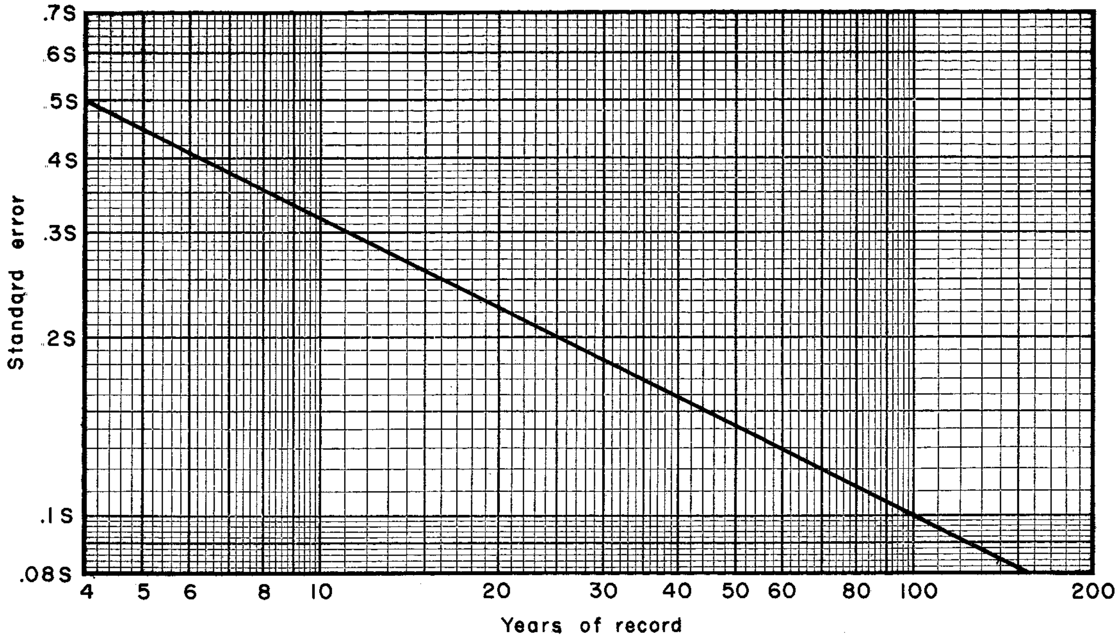


Fig. a. Standard error of a calculated mean.

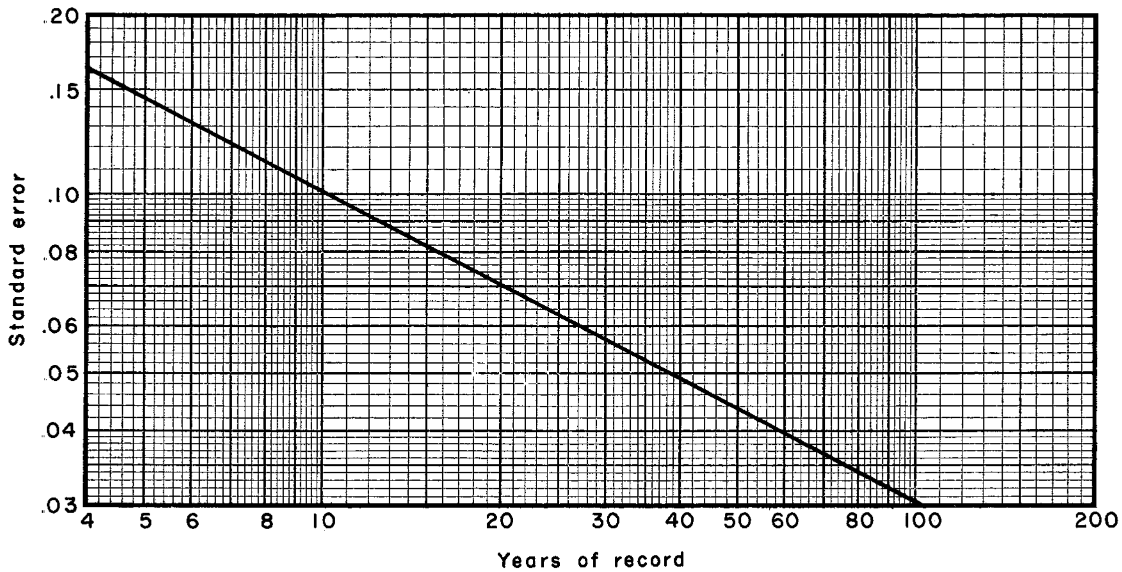
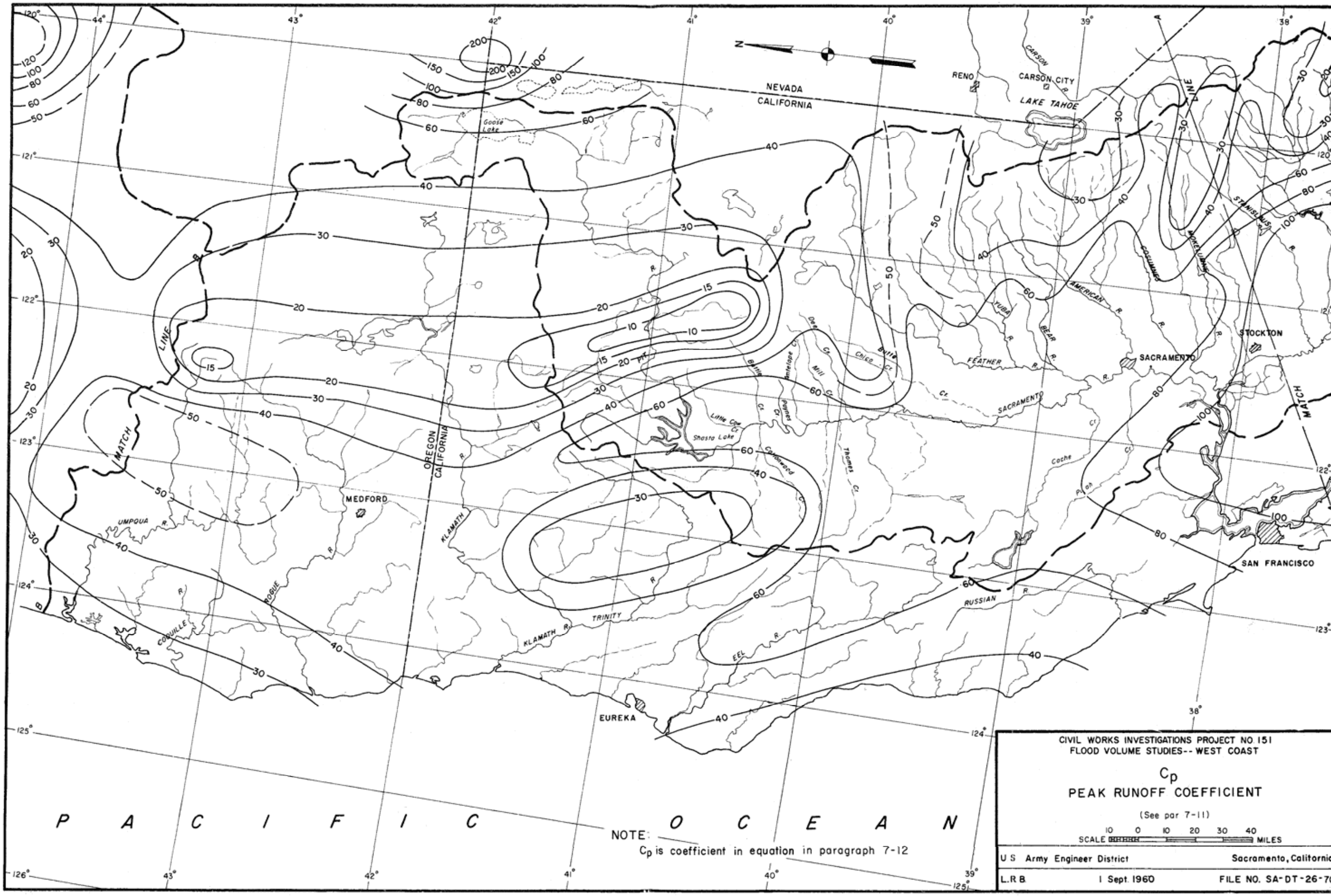


Fig. b. Approximate standard error of the logarithm of a calculated standard deviation

STANDARD ERRORS OF FREQUENCY STATISTICS

(See par. 10-02)



NOTE: C_p is coefficient in equation in paragraph 7-12

CIVIL WORKS INVESTIGATIONS PROJECT NO. 151
FLOOD VOLUME STUDIES-- WEST COAST

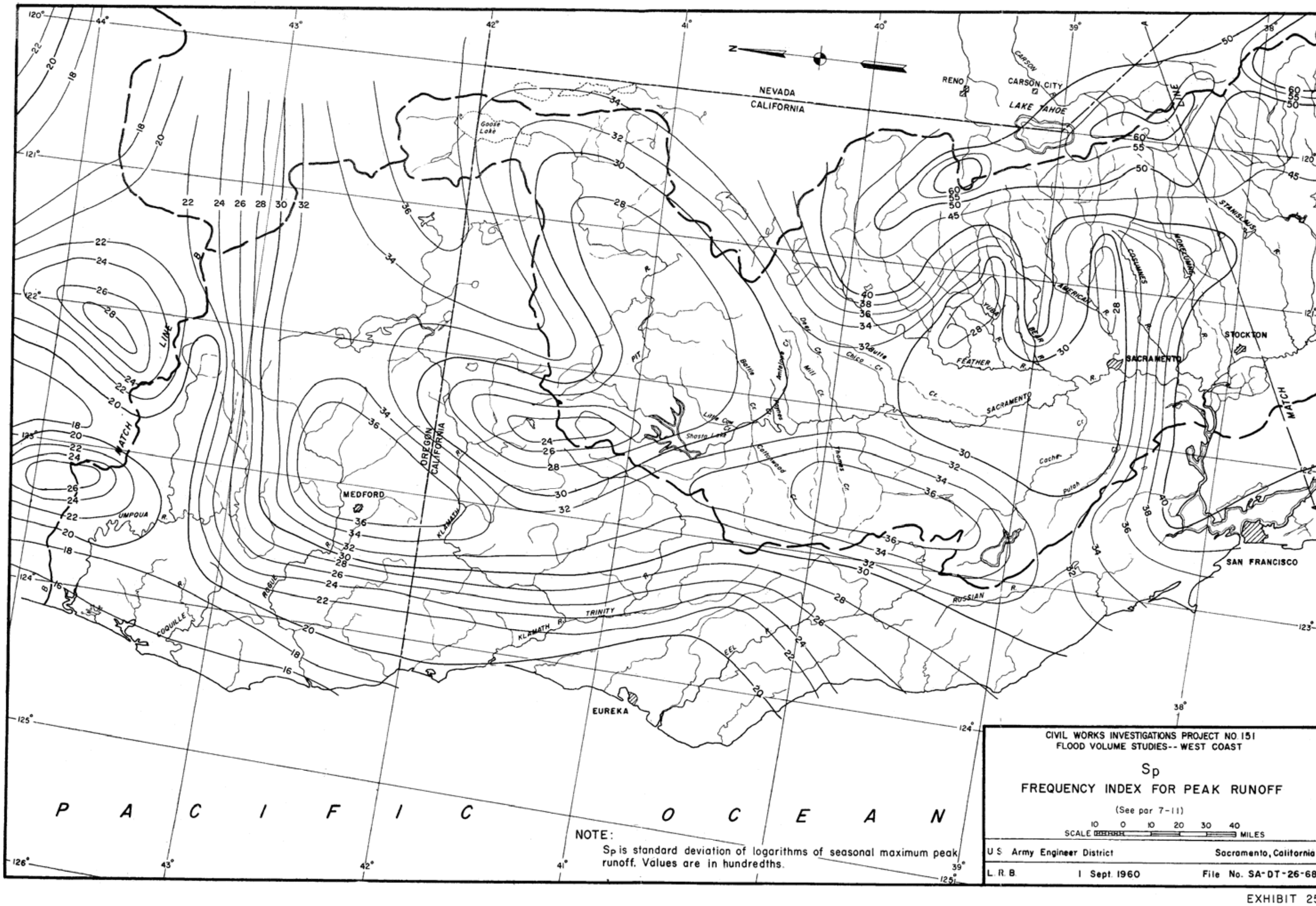
C_p
PEAK RUNOFF COEFFICIENT

(See par 7-11)

SCALE 10 0 10 20 30 40 MILES

U.S. Army Engineer District	Sacramento, California
L.R.B. 1 Sept. 1960	FILE NO. SA-DT-26-70

EXHIBIT 24



CIVIL WORKS INVESTIGATIONS PROJECT NO. 151
FLOOD VOLUME STUDIES-- WEST COAST

S_p

FREQUENCY INDEX FOR PEAK RUNOFF

(See par 7-11)

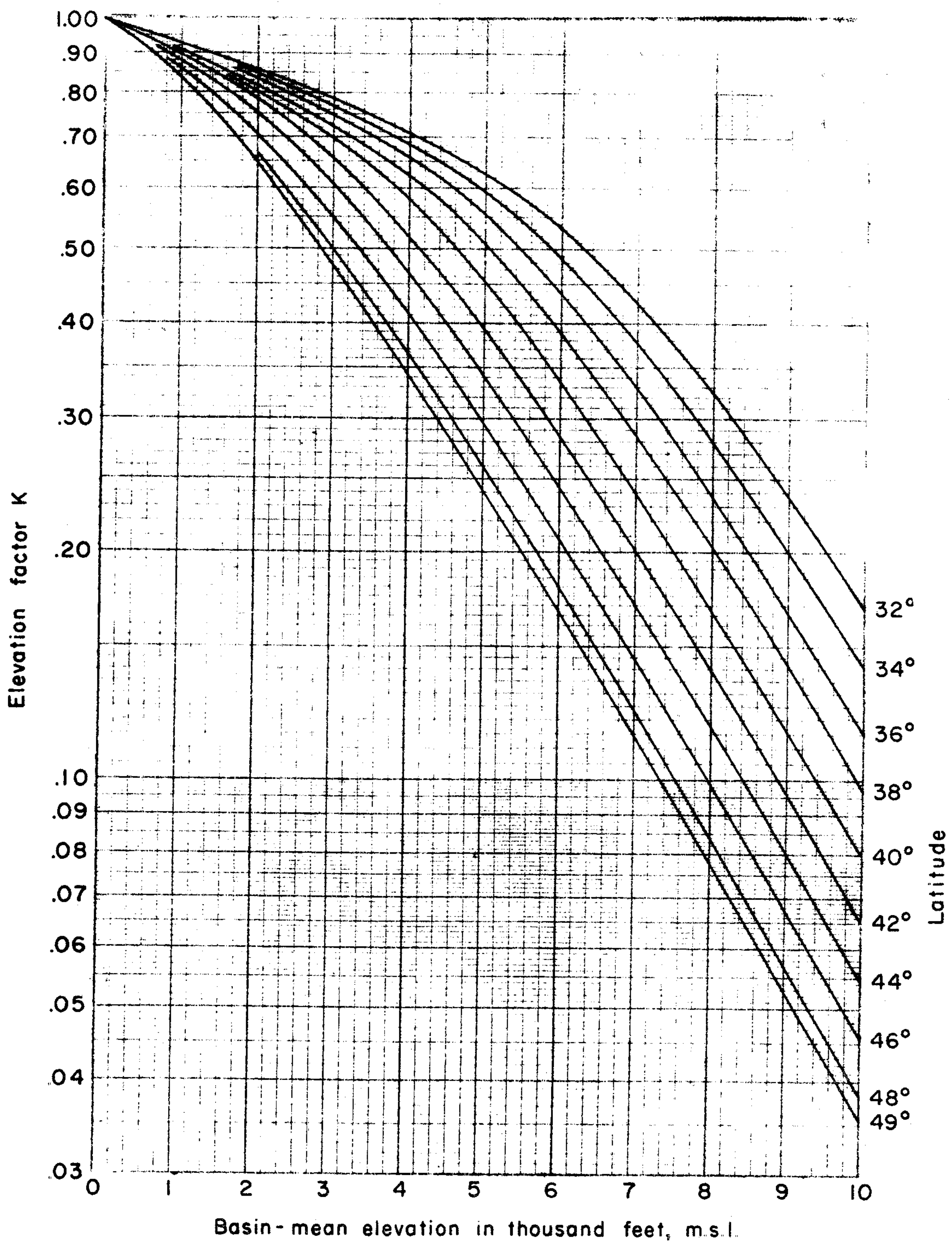
SCALE 10 0 10 20 30 40 MILES

U.S. Army Engineer District | Sacramento, California

L. R. B. | 1 Sept. 1960 | File No. SA-DT-26-68

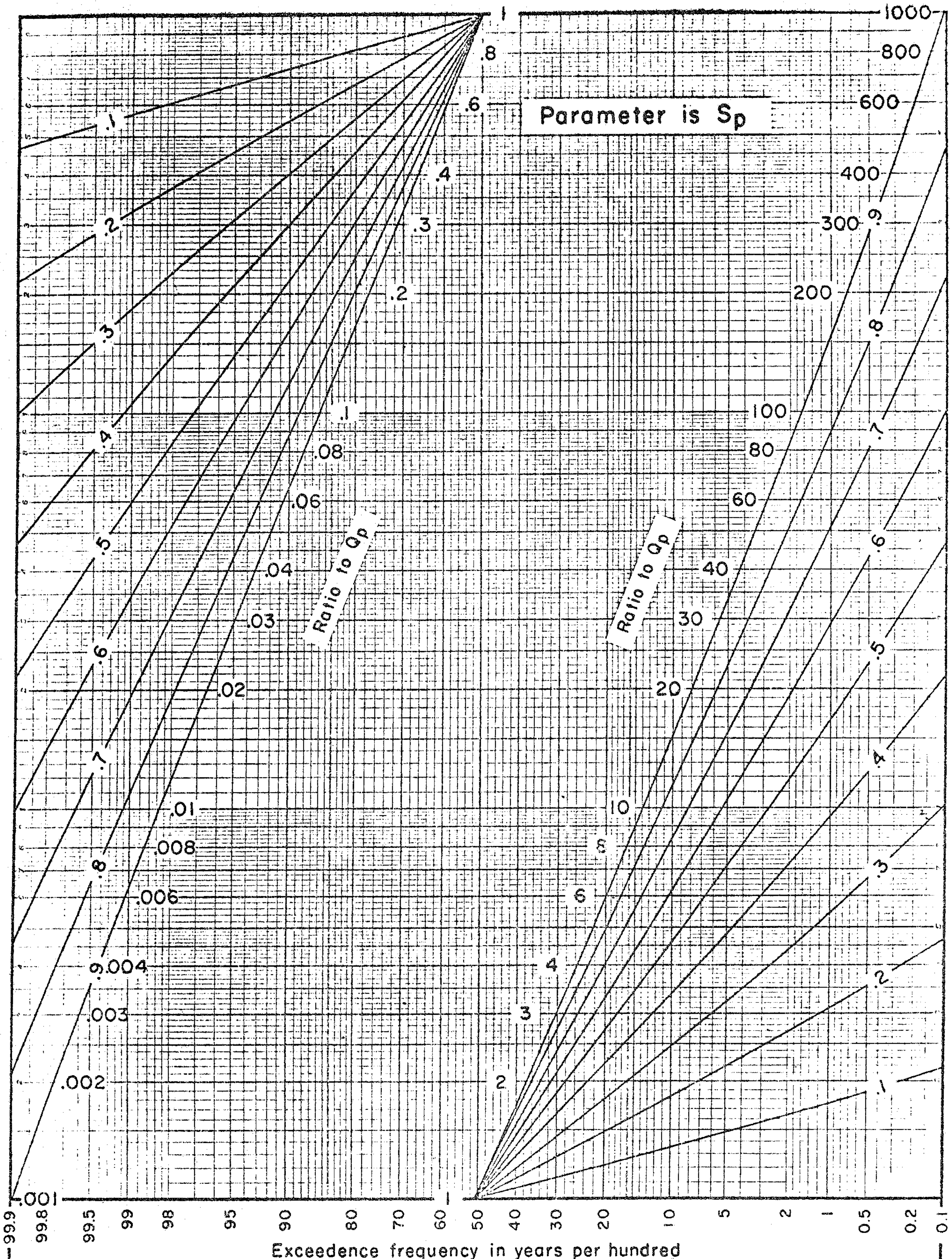
NOTE:
S_p is standard deviation of logarithms of seasonal maximum peak runoff. Values are in hundredths.

EXHIBIT 25



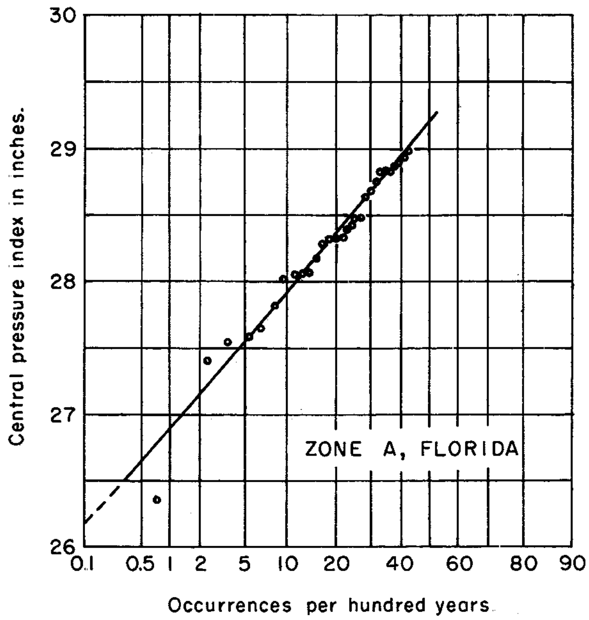
BASIN ELEVATION FACTOR

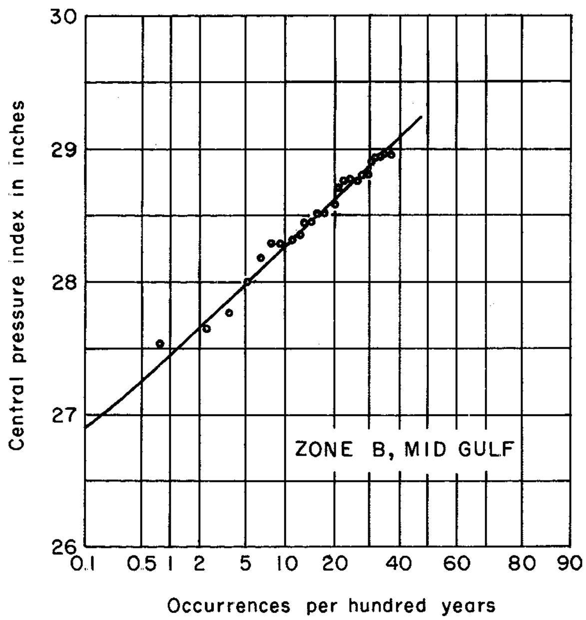
(See par. 7-11)

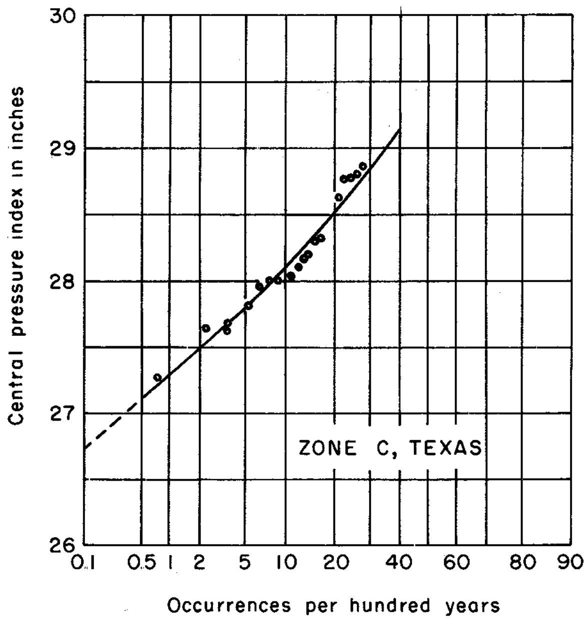


INDEX FREQUENCY CURVES FOR PEAK RUNOFF

(See par. 7-11)







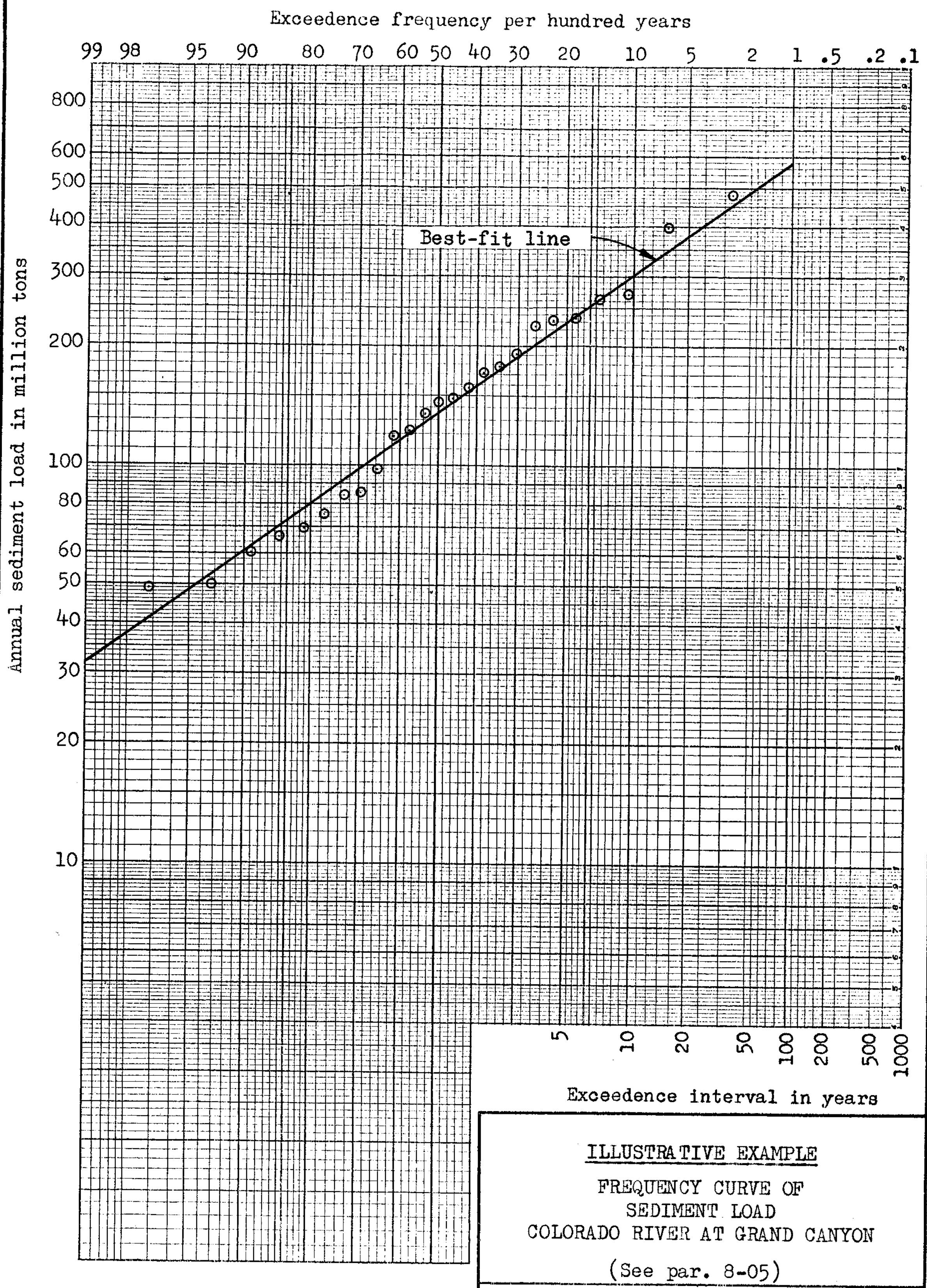
NOTE:

Drawings prepared for the Corps of Engineers by the U.S. Weather Bureau.

ILLUSTRATIVE EXAMPLE

ACCUMULATED FREQUENCY OF HURRICANE CENTRAL PRESSURES
(PLOTTED AS FREQUENCY PER 100 YEARS BASED ON 1900-1967)

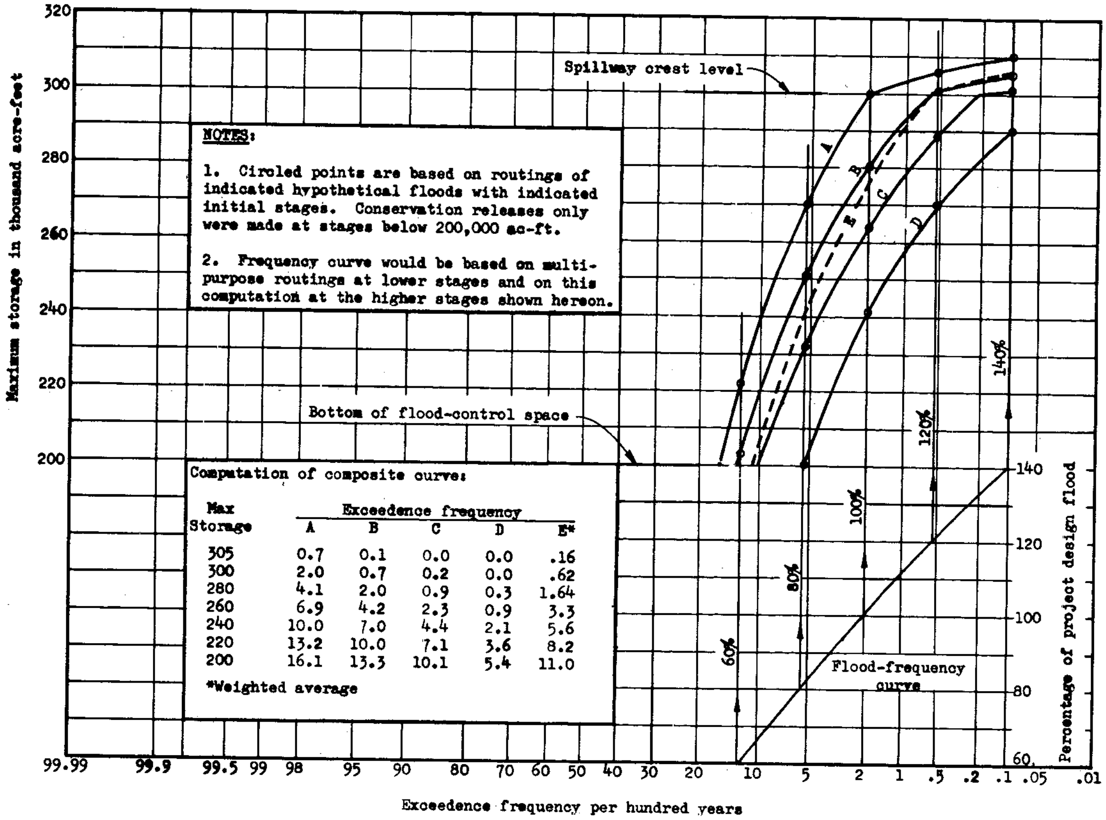
(See par. 8-04)



ILLUSTRATIVE EXAMPLE

FREQUENCY CURVE OF
SEDIMENT LOAD
COLORADO RIVER AT GRAND CANYON

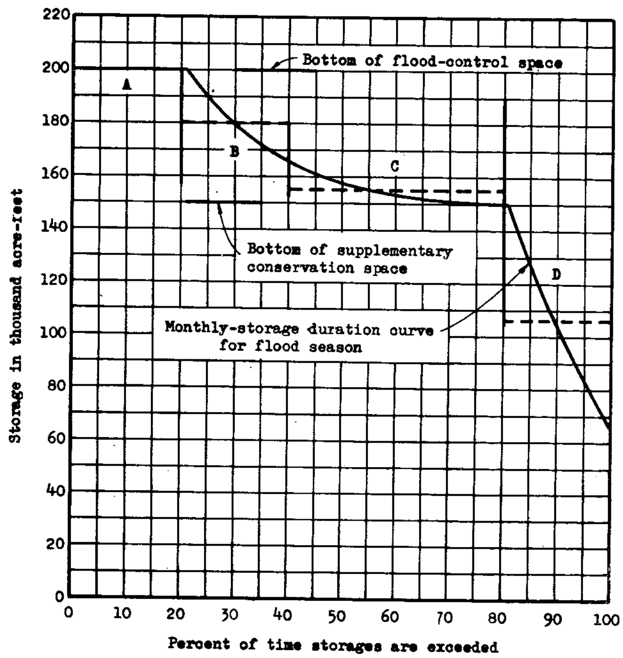
(See par. 8-05)



Computation of composite curve:

Max Storage	Exceedence frequency A	B	C	D	E*
305	0.7	0.1	0.0	0.0	.16
300	2.0	0.7	0.2	0.0	.62
280	4.1	2.0	0.9	0.3	1.64
260	6.9	4.2	2.3	0.9	3.3
240	10.0	7.0	4.4	2.1	5.6
220	13.2	10.0	7.1	3.6	8.2
200	16.1	13.3	10.1	5.4	11.0

*Weighted average



A - Initial storage, 200,000 ac-ft. (20% of time)
B - Initial storage, 180,000 ac-ft. (20% of time)
C - Initial storage, 155,000 ac-ft. (40% of time)
D - Initial storage, 106,000 ac-ft. (20% of time)

ILLUSTRATIVE EXAMPLE

STORAGE FREQUENCY COMPUTATION BASED ON COINCIDENT FREQUENCIES

(See par. 8-06)

ILLUSTRATIVE EXAMPLE

COMPUTATION OF SIMPLE LINEAR CORRELATION

(See par. 9-04)

Sta. (1)	Max. 12 hr (X_1') In. (2)	Max. 12 hr (X_1) Log. (3)	Mean Annual (X_2') In. (4)	Mean Annual (X_2) Log. (5)	Sta. (6)	Max. 12 hr (X_1') In. (7)	Max. 12 hr (X_1) Log. (8)	Mean Annual (X_2') In. (9)	Mean Annual (X_2) Log. (10)
7-0-12	3.1	.49	10.0	1.00	7-0-309	4.6	.66	19.2	1.28
7-0-15	7.5	.88	30.8	1.49	7-P-20	5.2	.72	13.3	1.12
7-0-22	12.6	1.10	42.2	1.63	7-P-25	3.1	.49	12.2	1.09
7-0-23	10.4	1.02	36.2	1.56	7-P-61	4.9	.69	12.2	1.09
7-0-36	4.9	.69	15.4	1.19	8-0-8	5.2	.72	25.2	1.40
7-0-39	7.1	.85	20.0	1.30	8-0-18	4.6	.66	18.5	1.27
7-0-43	5.3	.72	14.2	1.15	8-0-29	11.8	1.07	32.3	1.51
7-0-77	9.7	.99	24.5	1.39	8-0-34	12.4	1.09	33.8	1.53
7-0-84	6.2	.79	19.3	1.29	8-0-35	10.1	1.00	29.8	1.47
7-0-89	10.0	1.00	18.8	1.27	8-0-45	6.0	.78	19.7	1.29
7-0-93	6.0	.78	16.2	1.21	8-0-60	8.2	.91	22.5	1.35
7-0-95	5.8	.76	18.2	1.26	8-0-67	9.8	.99	28.7	1.46
7-0-99	4.5	.65	15.8	1.20	8-0-75	11.7	1.07	36.9	1.57
7-0-102	5.3	.72	17.3	1.24	8-0-219	10.0	1.00	27.8	1.44
7-0-110	9.3	.97	21.3	1.33	7-0-434	6.7	.83	23.1	1.36
7-0-114	8.5	.93	26.8	1.43					
7-0-120	11.6	1.06	29.0	1.46	N		42		42
7-0-122	10.5	1.02	28.8	1.46	ΣX		36.68		56.81
7-0-124	11.6	1.06	31.1	1.49	M		.873		1.353
7-0-125	9.9	1.00	27.9	1.45					
7-0-127	12.9	1.11	32.2	1.51	ΣX^2		33.2134		77.8041
7-0-130	13.4	1.13	44.6	1.65	$(\Sigma X)^2/N$		32.0339		76.8423
7-0-133	8.4	.92	23.8	1.38	Σx^2		1.1795		.9618
7-0-136	7.9	.90	27.2	1.43					
7-0-149	8.0	.90	19.2	1.28	$\Sigma(X_1X_2)$		50.5601		
7-0-182	5.2	.72	16.1	1.21	$\Sigma X_1 \Sigma X_2/N$		49.6141		
7-0-190	6.9	.84	20.9	1.32	$\Sigma(x_1x_2)$		.9460		

b_2 = .9460/.9618 = 0.98 (Eq. 20)

a = .873 - .98 (1.353) = -.453 (Eq. 21)

$R^2 = \dfrac{.98\,(.9460)}{1.1795} = 0.786$

$\bar{R}^2$ = 1 - (0.214) 41/40 = 0.78 (Eq. 31) $\bar{R}$ = 0.88

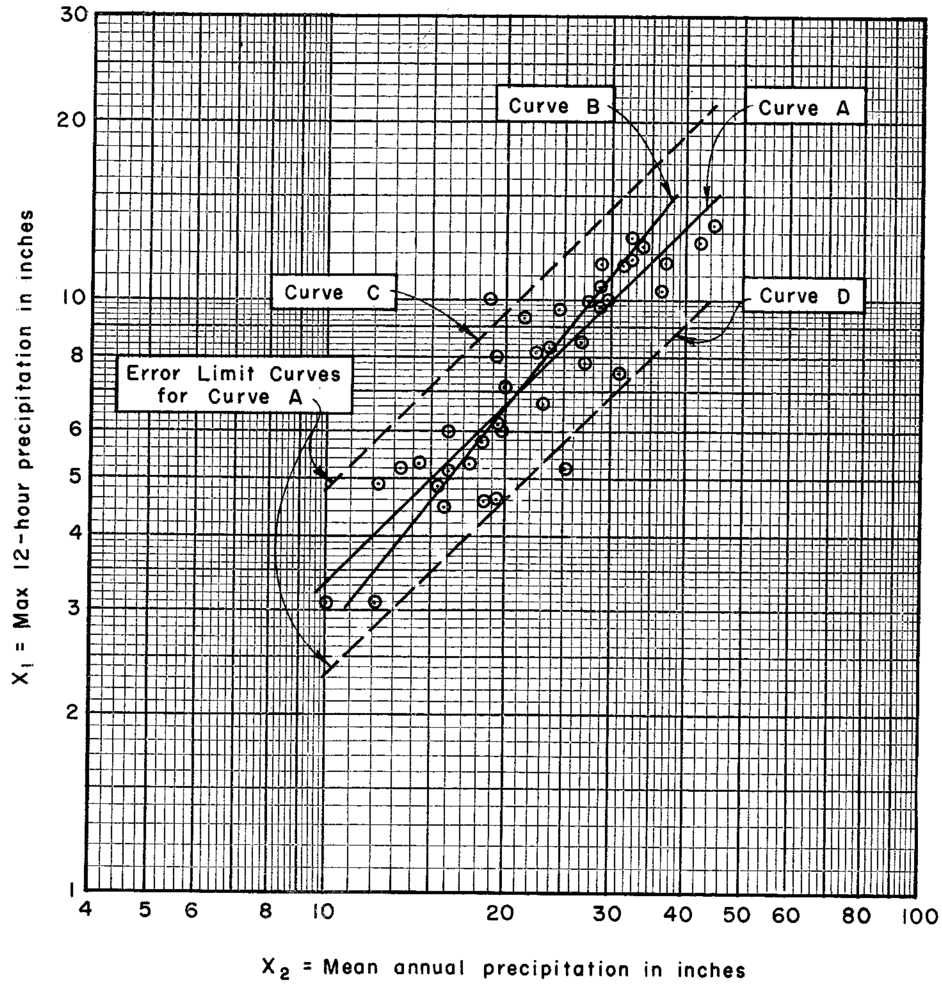
X_1 = $.98X_2$ - 0.45 (regression eq., Eq. 22)

log X_1' = .98 log X_2' - 0.45 = log $(.36\ X_2'^{.98})$

$X_1' = .36\ X_2'^{.98}$ (transformed regression Eq.)

S_e^2 = 0.22 (1.18)/41 = .0063 (Eq. 35)

S_e = .079 antilog 2 S_e = 1.44



NOTE:

See Exhibit 31 for computations
for Curve A.

ILLUSTRATION OF SIMPLE LINEAR CORRELATION

(See par. 9-04)

(See par. 9-06)

ILLUSTRATIVE EXAMPLE

COMPUTATION OF MULTIPLE LINEAR CORRELATION

Water year (1)	X_2 Log. Snow Cover (2)	X_3 Log. Ground Water (3)	X_4 Log. April Precip. (4)	X_1 Log. Runoff (5)
1936	.399	.325	.710	.939
1937	.343	.385	.634	.945
1938	.369	.408	.886	1.052
1939	.246	.428	.581	.744
1940	.181	.316	1.027	.666
1941	.297	.460	1.315	1.081
1942	.299	.511	1.097	1.060
1943	.354	.379	.707	.892
1944	.295	.395	1.240	1.021
1945	.321	.376	1.091	.920
1946	.168	.413	1.038	.755
1947	.280	.410	.979	.960
ΣX	3.552	4.806	11.305	11.035
M	.296	.400	.942	.920
$\Sigma(XX_2)$	1.1059	1.4197	3.2898	3.3365
$\Sigma X \Sigma X_2/N$	1.0514	1.4226	3.3463	3.2664
$\Sigma(xx_2)$	.0545	-.0029	-.0565	.0701
$\Sigma(XX_3)$		1.9558	4.5730	4.4587
$\Sigma X \Sigma X_3/N$		1.9248	4.5277	4.4195
$\Sigma(xx_3)$	-.0029	.0310	.0453	.0392
$\Sigma(XX_4)$			11.2796	10.5224
$\Sigma X \Sigma X_4/N$			10.6502	10.3959
$\Sigma(xx_4)$	-.0565	.0453	.6294	.1265
$\Sigma(XX_1)$				10.3468
$\Sigma X \Sigma X_1/N$				10.1476
$\Sigma(xx_1)$				.1992

$$\left.\begin{array}{ll} .0545\, b_2 - .0029\, b_3 - .0565\, b_4 = .0701 & \text{(Eq. 25)} \\ -.0029\, b_2 + .0310\, b_3 + .0453\, b_4 = .0392 & \text{(Eq. 26)} \\ -.0565\, b_2 + .0453\, b_3 + .6294\, b_4 = .1265 & \text{(Eq. 27)} \end{array}\right\} \begin{array}{l} b_2 = 1.623 \\ b_3 = 1.012 \\ b_4 = 0.274 \end{array}$$

$$a = .920 - 1.623\,(.296) - 1.012\,(.400) - .274\,(.942) = -.223 \quad \text{(Eq. 28)}$$

$$X_1 = 1.623\, X_2 + 1.012\, X_3 + 0.274\, X_4 - 0.223 \quad \text{(Regression equation)}$$

$$R^2 = \frac{1.623\,(.0701) + 1.012\,(.0392) + 0.274\,(.1265)}{.1992} = .944 \quad \text{(Eq. 32)}$$

$$\bar{R}^2 = 1 - (.056)11/8 = 0.923 \qquad \bar{R} = 0.96 \quad \text{(Eq. 31)}$$

$$S_e^2 = .077\,(.1992)/11 = 0.00139 \quad \text{(Eq. 35)}$$

$S_e = .037$ antilog $2\, S_e = 1.18$

VALUES OF t

d.f.	Exceedence probability										
	.45	.40	.35	.30	.25	.20	.15	.10	.05	.01	.005
1	.158	.325	.510	.727	1.000	1.376	1.963	3.078	6.314	31.821	63.657
2	.142	.289	.445	.617	.816	1.061	1.386	1.886	2.920	6.965	9.925
3	.137	.277	.424	.584	.765	.978	1.250	1.638	2.353	4.541	5.841
4	.134	.271	.414	.569	.741	.941	1.190	1.533	2.132	3.747	4.604
5	.132	.267	.408	.559	.727	.920	1.156	1.476	2.015	3.365	4.032
6	.131	.265	.404	.553	.718	.906	1.134	1.440	1.943	3.143	3.707
7	.130	.263	.402	.549	.711	.896	1.119	1.415	1.895	2.998	3.499
8	.130	.262	.399	.546	.706	.889	1.108	1.397	1.860	2.896	3.355
9	.129	.261	.398	.543	.703	.883	1.100	1.383	1.833	2.821	3.250
10	.129	.260	.397	.542	.700	.879	1.093	1.372	1.812	2.764	3.169
11	.129	.260	.396	.540	.697	.876	1.088	1.363	1.796	2.718	3.106
12	.128	.259	.395	.539	.695	.873	1.083	1.356	1.782	2.681	3.055
13	.128	.259	.394	.538	.694	.870	1.079	1.350	1.771	2.650	3.012
14	.128	.258	.393	.537	.692	.868	1.076	1.345	1.761	2.624	2.977
15	.128	.258	.393	.536	.691	.866	1.074	1.341	1.753	2.602	2.947
16	.128	.258	.392	.535	.690	.865	1.071	1.337	1.746	2.583	2.921
17	.128	.257	.392	.534	.689	.863	1.069	1.333	1.740	2.567	2.898
18	.127	.257	.392	.534	.688	.862	1.067	1.330	1.734	2.552	2.878
19	.127	.257	.391	.533	.688	.861	1.066	1.328	1.729	2.539	2.861
20	.127	.257	.391	.533	.687	.860	1.064	1.325	1.725	2.528	2.845
21	.127	.257	.391	.532	.686	.859	1.063	1.323	1.721	2.518	2.831
22	.127	.256	.390	.532	.686	.858	1.061	1.321	1.717	2.508	2.819
23	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.500	2.807
24	.127	.256	.390	.531	.685	.857	1.059	1.318	1.711	2.492	2.797
25	.127	.256	.390	.531	.684	.856	1.058	1.316	1.708	2.485	2.787
26	.127	.256	.390	.531	.684	.856	1.058	1.315	1.706	2.479	2.779
27	.127	.256	.389	.531	.684	.855	1.057	1.314	1.703	2.473	2.771
28	.127	.256	.389	.530	.683	.855	1.056	1.313	1.701	2.467	2.763
29	.127	.256	.389	.530	.683	.854	1.055	1.311	1.699	2.462	2.756
30	.127	.256	.389	.530	.683	.854	1.055	1.310	1.697	2.457	2.750
40	.126	.255	.388	.529	.681	.851	1.050	1.303	1.684	2.423	2.704
60	.126	.254	.387	.527	.679	.848	1.046	1.296	1.671	2.390	2.660
120	.126	.254	.386	.526	.677	.845	1.041	1.289	1.658	2.358	2.617
∞	.126	.253	.385	.524	.674	.842	1.036	1.282	1.645	2.326	2.576

VALUES OF χ^2

d.f.	Exceedence probability								
	.99	.95	.90	.70	.50	.30	.10	.05	.01
1	$.0^3157$	.00393	.0158	.148	.455	1.074	2.706	3.841	6.635
2	.0201	.103	.211	.713	1.386	2.408	4.605	5.991	9.210
3	.115	.352	.584	1.424	2.366	3.665	6.251	7.815	11.345
4	.297	.711	1.064	2.195	3.357	4.878	7.779	9.488	13.277
5	.554	1.145	1.610	3.000	4.351	6.064	9.236	11.070	15.086
6	.872	1.635	2.204	3.828	5.348	7.231	10.645	12.592	16.812
7	1.239	2.167	2.833	4.671	6.346	8.383	12.017	14.067	18.475
8	1.646	2.733	3.490	5.527	7.344	9.524	13.362	15.507	20.090
9	2.088	3.325	4.168	6.393	8.343	10.656	14.684	16.919	21.666
10	2.558	3.940	4.865	7.267	9.342	11.781	15.987	18.307	23.209
11	3.053	4.575	5.578	8.148	10.341	12.899	17.275	19.675	24.725
12	3.571	5.226	6.304	9.034	11.340	14.011	18.549	21.026	26.217
13	4.107	5.892	7.042	9.926	12.340	15.119	19.812	22.362	27.688
14	4.660	6.571	7.790	10.821	13.339	16.222	21.064	23.685	29.141
15	5.229	7.261	8.547	11.721	14.339	17.322	22.307	24.996	30.578
16	5.812	7.962	9.312	12.624	15.338	18.418	23.542	26.296	32.000
17	6.408	8.672	10.085	13.531	16.338	19.511	24.769	27.587	33.409
18	7.015	9.390	10.865	14.440	17.338	20.601	25.989	28.869	34.805
19	7.633	10.117	11.651	15.352	18.338	21.689	27.204	30.144	36.191
20	8.260	10.851	12.443	16.266	19.337	22.775	28.412	31.410	37.566
21	8.897	11.591	13.240	17.182	20.337	23.858	29.615	32.671	38.932
22	9.542	12.338	14.041	18.101	21.337	24.939	30.813	33.924	40.289
23	10.196	13.091	14.848	19.021	22.337	26.018	32.007	35.172	41.638
24	10.856	13.848	15.659	19.943	23.337	27.096	33.196	36.415	42.980
25	11.524	14.611	16.473	20.867	24.337	28.172	34.382	37.652	44.314
26	12.198	15.379	17.292	21.792	25.336	29.246	35.563	38.885	45.642
27	12.879	16.151	18.114	22.719	26.336	30.319	36.741	40.113	46.963
28	13.565	16.928	18.939	23.647	27.336	31.391	37.916	41.337	48.278
29	14.256	17.708	19.768	24.577	28.336	32.461	39.087	42.557	49.588
30	14.953	18.493	20.599	25.508	29.336	33.530	40.256	43.773	50.892

For higher degrees of freedom, values of $\sqrt{2\chi^2}$ are distributed approximately normally about $\sqrt{2(\text{d.f.})-1}$ with a standard deviation of 1.

TRANSFORMED PLOTTING POSITIONS (1)
k VALUES HAVING ZERO MEAN AND UNIT STANDARD DEVIATION
(See par. 4-05)

Event No. \ N	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
1	1.71	1.75	1.79	1.82	1.85	1.88	1.91	1.93	1.95	1.97	1.99	2.01	2.03	2.05	2.07	2.09	2.10	2.11	2.12	2.13	2.15	2.16	2.17	2.18	2.19
2	1.11	1.16	1.21	1.26	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.50	1.52	1.54	1.56	1.58	1.60	1.62	1.64	1.66	1.67	1.68	1.70	1.71	1.72
3	0.76	0.82	0.88	0.93	0.98	1.02	1.06	1.10	1.13	1.16	1.19	1.22	1.24	1.26	1.28	1.30	1.32	1.34	1.36	1.38	1.40	1.42	1.44	1.46	1.47
4	0.48	0.56	0.62	0.68	0.73	0.78	0.83	0.87	0.91	0.94	0.97	1.00	1.03	1.06	1.09	1.11	1.13	1.15	1.17	1.19	1.21	1.23	1.25	1.27	1.29
5	0.25	0.32	0.40	0.47	0.53	0.59	0.64	0.68	0.72	0.76	0.80	0.83	0.86	0.89	0.92	0.95	0.97	0.99	1.01	1.04	1.06	1.08	1.10	1.12	1.14
6	0	0.11	0.20	0.27	0.34	0.41	0.47	0.52	0.56	0.60	0.64	0.68	0.71	0.74	0.77	0.80	0.83	0.86	0.88	0.90	0.93	0.95	0.97	0.99	1.01
7			0	0.09	0.17	0.24	0.30	0.36	0.41	0.46	0.50	0.54	0.58	0.61	0.64	0.67	0.70	0.73	0.76	0.79	0.81	0.83	0.85	0.87	0.89
8					0	0.08	0.15	0.21	0.27	0.32	0.37	0.41	0.45	0.49	0.52	0.55	0.58	0.61	0.64	0.67	0.70	0.73	0.75	0.77	0.79
9							0	0.07	0.13	0.19	0.24	0.29	0.34	0.38	0.42	0.45	0.48	0.51	0.54	0.57	0.60	0.63	0.65	0.67	0.70
10									0	0.07	0.12	0.17	0.22	0.27	0.31	0.35	0.38	0.41	0.44	0.47	0.50	0.53	0.56	0.59	0.61
11											0	0.06	0.11	0.16	0.20	0.24	0.28	0.32	0.35	0.38	0.41	0.44	0.47	0.50	0.52
12													0	0.05	0.10	0.15	0.19	0.23	0.26	0.30	0.33	0.36	0.39	0.42	0.45
13															0	0.05	0.10	0.14	0.17	0.21	0.24	0.28	0.31	0.34	0.37
14																	0	0.06	0.10	0.13	0.16	0.20	0.23	0.26	0.29
15																			0	0.04	0.08	0.12	0.15	0.18	0.22
16																					0	0.04	0.08	0.11	0.14
17																							0	0.04	0.07
18																									0

Event No. \ N	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
1	2.20	2.22	2.23	2.24	2.25	2.26	2.27	2.28	2.28	2.29	2.30	2.31	2.31	2.32	2.33	2.33	2.34	2.35	2.36	2.37	2.37	2.38	2.39	2.39	2.40
2	1.74	1.75	1.76	1.77	1.79	1.80	1.81	1.82	1.83	1.84	1.85	1.86	1.87	1.88	1.88	1.89	1.90	1.91	1.92	1.93	1.93	1.94	1.95	1.96	1.97
3	1.48	1.50	1.51	1.53	1.54	1.55	1.56	1.57	1.59	1.60	1.61	1.62	1.63	1.64	1.65	1.66	1.67	1.68	1.69	1.69	1.70	1.71	1.72	1.73	1.74
4	1.30	1.31	1.33	1.35	1.36	1.37	1.39	1.40	1.41	1.42	1.43	1.44	1.45	1.47	1.48	1.49	1.50	1.51	1.52	1.53	1.54	1.55	1.56	1.56	1.57
5	1.15	1.17	1.19	1.20	1.22	1.23	1.25	1.26	1.27	1.28	1.30	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39	1.40	1.41	1.42	1.43	1.44
6	1.03	1.05	1.06	1.08	1.10	1.11	1.12	1.14	1.15	1.17	1.18	1.19	1.20	1.21	1.22	1.24	1.25	1.26	1.27	1.28	1.29	1.30	1.31	1.32	1.33
7	0.91	0.93	0.95	0.97	0.99	1.01	1.02	1.04	1.05	1.06	1.08	1.09	1.10	1.12	1.13	1.14	1.15	1.16	1.18	1.19	1.20	1.21	1.22	1.23	1.24
8	0.81	0.83	0.85	0.87	0.89	0.91	0.92	0.94	0.95	0.97	0.98	1.00	1.01	1.03	1.04	1.05	1.06	1.07	1.09	1.10	1.11	1.12	1.13	1.14	1.15
9	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.85	0.87	0.88	0.90	0.91	0.93	0.94	0.96	0.97	0.98	0.99	1.00	1.02	1.03	1.04	1.05	1.06	1.07
10	0.63	0.65	0.67	0.70	0.72	0.74	0.75	0.77	0.79	0.80	0.82	0.84	0.85	0.86	0.88	0.89	0.91	0.92	0.93	0.94	0.96	0.97	0.98	0.99	1.00
11	0.55	0.57	0.59	0.62	0.64	0.66	0.68	0.70	0.71	0.73	0.75	0.76	0.78	0.79	0.80	0.82	0.84	0.85	0.86	0.88	0.89	0.90	0.91	0.92	0.94
12	0.47	0.49	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.69	0.71	0.72	0.74	0.75	0.77	0.78	0.80	0.81	0.82	0.84	0.85	0.86	0.87
13	0.39	0.42	0.45	0.47	0.49	0.51	0.53	0.55	0.57	0.59	0.61	0.63	0.64	0.66	0.68	0.69	0.71	0.72	0.74	0.75	0.76	0.78	0.79	0.80	0.81
14	0.32	0.35	0.37	0.40	0.42	0.44	0.46	0.49	0.51	0.53	0.54	0.56	0.58	0.60	0.61	0.63	0.64	0.66	0.68	0.69	0.70	0.72	0.73	0.74	0.76
15	0.25	0.27	0.30	0.33	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.57	0.59	0.60	0.62	0.63	0.65	0.66	0.68	0.69	0.70
16	0.17	0.21	0.24	0.26	0.29	0.31	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50	0.51	0.53	0.55	0.56	0.58	0.59	0.61	0.62	0.64	0.65
17	0.11	0.14	0.17	0.20	0.22	0.25	0.27	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.49	0.51	0.52	0.54	0.56	0.57	0.58	0.60
18	0.04	0.07	0.10	0.13	0.16	0.19	0.21	0.24	0.26	0.28	0.31	0.33	0.35	0.37	0.39	0.40	0.42	0.44	0.46	0.47	0.49	0.51	0.52	0.53	0.55
19		0	0.03	0.06	0.09	0.12	0.15	0.18	0.20	0.22	0.25	0.27	0.29	0.31	0.33	0.35	0.37	0.39	0.41	0.42	0.44	0.46	0.47	0.49	0.50
20				0	0.03	0.06	0.09	0.12	0.14	0.17	0.19	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.37	0.39	0.41	0.42	0.44	0.45
21						0	0.03	0.06	0.09	0.11	0.14	0.16	0.18	0.21	0.23	0.25	0.27	0.29	0.31	0.32	0.34	0.36	0.38	0.39	0.41
22								0	0.03	0.06	0.08	0.11	0.13	0.15	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.31	0.33	0.35	0.36
23										0	0.03	0.05	0.08	0.10	0.13	0.15	0.17	0.19	0.21	0.23	0.25	0.27	0.29	0.30	0.32
24												0	0.03	0.05	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28
25														0	0.03	0.05	0.07	0.10	0.12	0.14	0.16	0.18	0.20	0.21	0.23
26																0	0.03	0.05	0.07	0.09	0.11	0.13	0.15	0.17	0.19
27																		0	0.03	0.05	0.07	0.09	0.11	0.13	0.15
28																				0	0.02	0.04	0.07	0.09	0.11
29																						0	0.02	0.04	0.06
30																								0	0.02

Event No. \ N	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85
1	2.41	2.41	2.42	2.42	2.43	2.43	2.44	2.44	2.45	2.45	2.46	2.46	2.47	2.47	2.48	2.48	2.49	2.49	2.50	2.50	2.51	2.51	2.51	2.52	2.52
2	1.97	1.98	1.99	1.99	2.00	2.01	2.01	2.02	2.02	2.03	2.04	2.04	2.05	2.05	2.06	2.06	2.07	2.07	2.08	2.08	2.09	2.09	2.10	2.10	2.11
3	1.74	1.75	1.76	1.76	1.77	1.78	1.79	1.79	1.80	1.81	1.81	1.82	1.83	1.83	1.84	1.84	1.85	1.85	1.86	1.86	1.87	1.87	1.88	1.88	1.89
4	1.58	1.59	1.60	1.60	1.61	1.62	1.63	1.63	1.64	1.65	1.66	1.66	1.67	1.68	1.68	1.69	1.69	1.70	1.71	1.71	1.72	1.72	1.73	1.73	1.74
5	1.45	1.46	1.47	1.48	1.48	1.49	1.50	1.51	1.52	1.52	1.53	1.54	1.55	1.55	1.56	1.57	1.57	1.58	1.59	1.59	1.60	1.60	1.61	1.61	1.62
6	1.34	1.35	1.36	1.37	1.38	1.39	1.39	1.40	1.41	1.42	1.43	1.43	1.44	1.45	1.45	1.46	1.47	1.48	1.48	1.49	1.49	1.50	1.50	1.51	1.52
7	1.25	1.26	1.27	1.28	1.29	1.29	1.30	1.31	1.32	1.33	1.33	1.34	1.35	1.36	1.36	1.37	1.38	1.39	1.39	1.40	1.40	1.41	1.42	1.42	1.43
8	1.16	1.17	1.18	1.19	1.20	1.21	1.22	1.23	1.24	1.24	1.25	1.26	1.27	1.28	1.28	1.29	1.30	1.31	1.31	1.32	1.33	1.33	1.34	1.35	1.35
9	1.09	1.10	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.18	1.19	1.20	1.20	1.21	1.22	1.23	1.23	1.24	1.25	1.26	1.26	1.27	1.27	1.28
10	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.18	1.19	1.20	1.20	1.21	1.22
11	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.07	1.08	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.14	1.15	1.16
12	0.88	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.06	1.06	1.07	1.08	1.09	1.10	1.10
13	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.01	1.02	1.03	1.04	1.04	1.05
14	0.77	0.78	0.79	0.80	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.94	0.95	0.96	0.97	0.98	0.99	0.99	1.00
15	0.71	0.73	0.74	0.75	0.76	0.77	0.79	0.80	0.81	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.90	0.91	0.91	0.92	0.93	0.94	0.94	0.95
16	0.66	0.68	0.69	0.70	0.71	0.72	0.73	0.75	0.76	0.77	0.78	0.79	0.80	0.81	0.82	0.83	0.84	0.85	0.86	0.86	0.87	0.88	0.89	0.90	0.91
17	0.61	0.63	0.64	0.65	0.66	0.68	0.69	0.70	0.71	0.72	0.73	0.74	0.75	0.77	0.78	0.78	0.79	0.80	0.81	0.82	0.83	0.84	0.85	0.86	0.86
18	0.56	0.58	0.59	0.60	0.62	0.63	0.64	0.65	0.66	0.68	0.69	0.70	0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.80	0.81	0.81	0.82
19	0.52	0.53	0.54	0.56	0.57	0.58	0.59	0.61	0.62	0.63	0.64	0.65	0.67	0.68	0.69	0.70	0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.77	0.78
20	0.47	0.48	0.50	0.51	0.53	0.54	0.55	0.56	0.58	0.59	0.60	0.61	0.62	0.63	0.64	0.66	0.67	0.68	0.69	0.70	0.71	0.72	0.73	0.73	0.74
21	0.42	0.44	0.45	0.47	0.48	0.50	0.51	0.52	0.53	0.55	0.56	0.57	0.58	0.59	0.60	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.69	0.70
22	0.38	0.39	0.41	0.42	0.44	0.45	0.47	0.48	0.49	0.50	0.52	0.53	0.54	0.55	0.56	0.58	0.59	0.60	0.61	0.62	0.63	0.64	0.65	0.66	0.67
23	0.34	0.35	0.37	0.38	0.40	0.41	0.42	0.44	0.45	0.46	0.48	0.49	0.50	0.51	0.52	0.54	0.55	0.56	0.57	0.58	0.59	0.60	0.61	0.62	0.63
24	0.29	0.31	0.33	0.34	0.35	0.37	0.38	0.40	0.41	0.42	0.44	0.45	0.46	0.48	0.49	0.50	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59
25	0.25	0.27	0.28	0.30	0.31	0.33	0.34	0.36	0.37	0.39	0.40	0.41	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.51	0.52	0.53	0.54	0.55	0.56
26	0.21	0.23	0.24	0.26	0.27	0.29	0.30	0.32	0.33	0.35	0.36	0.38	0.39	0.40	0.41	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50	0.51	0.52
27	0.17	0.18	0.20	0.22	0.23	0.25	0.27	0.28	0.30	0.31	0.32	0.34	0.35	0.36	0.38	0.39	0.40	0.41	0.42	0.44	0.45	0.46	0.47	0.48	0.49
28	0.13	0.14	0.16	0.18	0.19	0.21	0.23	0.24	0.26	0.27	0.29	0.30	0.32	0.33	0.34	0.36	0.37	0.38	0.39	0.40	0.41	0.43	0.44	0.45	0.46
29	0.08	0.10	0.12	0.14	0.16	0.17	0.19	0.20	0.22	0.24	0.25	0.27	0.28	0.29	0.31	0.32	0.33	0.34	0.36	0.37	0.38	0.39	0.40	0.41	0.43
30	0.04	0.06	0.08	0.10	0.12	0.13	0.15	0.17	0.18	0.20	0.21	0.23	0.24	0.26	0.27	0.29	0.30	0.31	0.32	0.33	0.35	0.36	0.37	0.38	0.39
31	0	0.02	0.04	0.06	0.08	0.10	0.11	0.13	0.15	0.16	0.18	0.19	0.21	0.22	0.24	0.25	0.26	0.28	0.29	0.30	0.32	0.33	0.34	0.35	0.36
32			0	0.02	0.04	0.06	0.08	0.09	0.11	0.13	0.14	0.16	0.17	0.19	0.20	0.22	0.23	0.24	0.26	0.27	0.28	0.30	0.31	0.32	0.33
33					0	0.02	0.04	0.06	0.07	0.09	0.11	0.12	0.14	0.15	0.17	0.18	0.20	0.21	0.22	0.24	0.25	0.26	0.28	0.29	0.30
34							0	0.02	0.04	0.06	0.07	0.09	0.11	0.12	0.14	0.15	0.16	0.18	0.19	0.21	0.22	0.23	0.24	0.26	0.27
35									0	0.02	0.04	0.05	0.07	0.09	0.10	0.12	0.13	0.15	0.16	0.17	0.19	0.20	0.21	0.23	0.24
36											0	0.02	0.04	0.05	0.07	0.08	0.10	0.11	0.13	0.14	0.16	0.17	0.18	0.20	0.21
37													0	0.02	0.03	0.05	0.07	0.08	0.10	0.11	0.12	0.14	0.15	0.17	0.18
38															0	0.02	0.03	0.05	0.06	0.08	0.09	0.11	0.12	0.14	0.15
39																	0	0.02	0.03	0.05	0.06	0.08	0.09	0.11	0.12
40																			0	0.02	0.03	0.05	0.06	0.08	0.09
41																					0	0.02	0.03	0.05	0.06
42																							0	0.02	0.03
43																									0

NOTE:

Same footnotes as sheet 2

TRANSFORMED PLOTTING POSITIONS (1)
k VALUES HAVING ZERO MEAN AND UNIT STANDARD DEVIATION
(See par. 4-05)

Event No. \ N	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2.53	2.53	2.53	2.54	2.54	2.55	2.55	2.55	2.56	2.56	2.56	2.57	2.57	2.57	2.58
2	2.11	2.12	2.12	2.13	2.13	2.14	2.14	2.14	2.15	2.15	2.16	2.16	2.16	2.17	2.17
3	1.89	1.90	1.90	1.91	1.91	1.92	1.92	1.93	1.93	1.94	1.94	1.95	1.95	1.96	1.96
4	1.74	1.75	1.75	1.76	1.77	1.77	1.78	1.78	1.79	1.79	1.80	1.80	1.80	1.81	1.81
5	1.62	1.63	1.63	1.64	1.65	1.65	1.66	1.66	1.67	1.67	1.68	1.68	1.69	1.69	1.70
6	1.52	1.53	1.53	1.54	1.55	1.55	1.56	1.56	1.57	1.58	1.58	1.59	1.59	1.60	1.60
7	1.44	1.44	1.45	1.45	1.46	1.47	1.47	1.48	1.48	1.49	1.49	1.50	1.50	1.51	1.51
8	1.36	1.37	1.37	1.38	1.38	1.39	1.40	1.40	1.41	1.41	1.42	1.42	1.43	1.43	1.44
9	1.29	1.30	1.30	1.31	1.32	1.32	1.33	1.33	1.34	1.34	1.35	1.36	1.36	1.37	1.37
10	1.22	1.23	1.24	1.25	1.25	1.26	1.26	1.27	1.28	1.28	1.29	1.29	1.30	1.30	1.31
11	1.17	1.17	1.18	1.19	1.19	1.20	1.21	1.21	1.22	1.22	1.23	1.24	1.24	1.25	1.25
12	1.11	1.12	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.17	1.18	1.18	1.19	1.19	1.20
13	1.06	1.07	1.07	1.08	1.09	1.09	1.10	1.11	1.11	1.12	1.12	1.13	1.14	1.14	1.15
14	1.01	1.02	1.02	1.03	1.04	1.04	1.05	1.06	1.07	1.07	1.08	1.09	1.09	1.10	1.10
15	0.96	0.97	0.98	0.98	0.99	1.00	1.01	1.01	1.02	1.03	1.03	1.04	1.05	1.05	1.06
16	0.92	0.92	0.93	0.94	0.95	0.95	0.96	0.97	0.98	0.98	0.99	1.00	1.00	1.01	1.01
17	0.87	0.88	0.89	0.90	0.91	0.91	0.92	0.93	0.93	0.94	0.95	0.95	0.96	0.97	0.97
18	0.83	0.84	0.85	0.86	0.86	0.87	0.88	0.89	0.89	0.90	0.91	0.92	0.92	0.93	0.93
19	0.79	0.80	0.81	0.82	0.82	0.83	0.84	0.85	0.85	0.86	0.87	0.88	0.88	0.89	0.90
20	0.75	0.76	0.77	0.78	0.78	0.79	0.80	0.81	0.82	0.82	0.83	0.84	0.85	0.85	0.86
21	0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.77	0.78	0.79	0.80	0.81	0.81	0.82	0.82
22	0.68	0.69	0.69	0.70	0.71	0.72	0.73	0.74	0.75	0.75	0.76	0.77	0.78	0.78	0.79
23	0.64	0.65	0.66	0.67	0.68	0.69	0.70	0.70	0.71	0.72	0.73	0.73	0.74	0.75	0.76
24	0.60	0.61	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.68	0.69	0.70	0.71	0.71	0.72
25	0.57	0.58	0.59	0.60	0.61	0.62	0.63	0.63	0.64	0.65	0.66	0.67	0.68	0.68	0.69
26	0.53	0.55	0.56	0.57	0.57	0.58	0.59	0.60	0.61	0.62	0.63	0.64	0.64	0.65	0.66
27	0.50	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.60	0.60	0.61	0.62	0.63
28	0.47	0.48	0.49	0.50	0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.57	0.58	0.59	0.60
29	0.44	0.45	0.46	0.47	0.48	0.49	0.50	0.51	0.52	0.53	0.54	0.54	0.55	0.56	0.57
30	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50	0.51	0.51	0.52	0.53	0.54
31	0.37	0.39	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.49	0.50	0.51
32	0.34	0.35	0.36	0.38	0.39	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.46	0.47	0.48
33	0.31	0.32	0.33	0.35	0.36	0.37	0.38	0.39	0.40	0.41	0.42	0.43	0.44	0.45	0.46
34	0.28	0.29	0.30	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40	0.41	0.42	0.43
35	0.25	0.26	0.27	0.29	0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40
36	0.22	0.23	0.24	0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37
37	0.19	0.20	0.22	0.23	0.24	0.25	0.26	0.27	0.29	0.30	0.31	0.32	0.33	0.34	0.35
38	0.16	0.18	0.19	0.20	0.21	0.22	0.23	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
39	0.13	0.15	0.16	0.17	0.18	0.19	0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29
40	0.10	0.12	0.13	0.14	0.16	0.17	0.18	0.19	0.20	0.21	0.23	0.24	0.25	0.26	0.27
41	0.07	0.09	0.10	0.11	0.13	0.14	0.15	0.16	0.18	0.19	0.20	0.21	0.22	0.23	0.24
42	0.04	0.06	0.07	0.09	0.10	0.11	0.12	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21
43	0.02	0.03	0.04	0.06	0.07	0.08	0.10	0.11	0.12	0.13	0.15	0.16	0.17	0.18	0.19
44		0	0.02	0.03	0.04	0.06	0.07	0.08	0.10	0.11	0.12	0.13	0.14	0.15	0.16
45				0	0.02	0.03	0.04	0.06	0.07	0.08	0.09	0.10	0.12	0.13	0.14
46						0	0.02	0.03	0.04	0.05	0.07	0.08	0.09	0.10	0.10
47								0	0.02	0.03	0.04	0.05	0.06	0.08	0.09
48										0	0.02	0.03	0.04	0.05	0.06
49												0	0.01	0.03	0.04
50														0	0.01

Event No. \ N	110	120	130	140	150	160	170	180	190	200
1	2.61	2.64	2.67	2.69	2.71	2.73	2.75	2.77	2.79	2.81
2	2.21	2.24	2.27	2.30	2.32	2.35	2.37	2.39	2.41	2.43
3	2.00	2.04	2.07	2.10	2.13	2.15	2.18	2.20	2.22	2.24
4	1.85	1.89	1.93	1.96	1.99	2.02	2.04	2.07	2.09	2.11
5	1.74	1.78	1.82	1.85	1.88	1.91	1.94	1.96	1.98	2.00
6	1.64	1.69	1.72	1.76	1.79	1.82	1.85	1.87	1.90	1.92
7	1.56	1.61	1.64	1.68	1.71	1.74	1.77	1.80	1.82	1.85
8	1.49	1.53	1.57	1.61	1.65	1.68	1.70	1.73	1.76	1.78
9	1.42	1.47	1.51	1.55	1.58	1.62	1.65	1.67	1.70	1.72
10	1.36	1.41	1.45	1.49	1.53	1.56	1.59	1.62	1.65	1.67
11	1.31	1.36	1.40	1.44	1.48	1.51	1.54	1.57	1.60	1.62
12	1.26	1.31	1.35	1.39	1.43	1.46	1.49	1.52	1.55	1.58
13	1.21	1.26	1.30	1.34	1.38	1.42	1.45	1.48	1.51	1.53
14	1.16	1.21	1.26	1.30	1.34	1.38	1.41	1.44	1.47	1.49
15	1.12	1.17	1.22	1.26	1.30	1.34	1.37	1.40	1.43	1.46
16	1.08	1.13	1.18	1.22	1.26	1.30	1.33	1.36	1.39	1.42
17	1.04	1.09	1.14	1.19	1.23	1.26	1.30	1.33	1.36	1.39
18	1.00	1.05	1.11	1.15	1.19	1.23	1.27	1.30	1.33	1.36
19	0.96	1.02	1.07	1.12	1.16	1.20	1.23	1.27	1.30	1.33
20	0.93	0.98	1.04	1.08	1.13	1.17	1.20	1.24	1.27	1.30
21	0.89	0.95	1.00	1.05	1.10	1.14	1.17	1.21	1.24	1.27
22	0.86	0.92	0.97	1.02	1.07	1.11	1.14	1.18	1.21	1.24
23	0.83	0.89	0.94	0.99	1.04	1.08	1.12	1.15	1.18	1.21
24	0.79	0.86	0.91	0.96	1.01	1.05	1.09	1.12	1.16	1.19
25	0.76	0.83	0.88	0.95	0.98	1.02	1.06	1.10	1.13	1.16
26	0.73	0.80	0.86	0.91	0.95	1.00	1.04	1.07	1.11	1.14
27	0.70	0.77	0.83	0.88	0.93	0.97	1.01	1.05	1.08	1.11
28	0.67	0.74	0.80	0.85	0.90	0.95	0.99	1.02	1.06	1.09
29	0.65	0.71	0.77	0.83	0.88	0.92	0.96	1.00	1.04	1.07
30	0.62	0.69	0.75	0.80	0.85	0.90	0.94	0.98	1.01	1.05
31	0.59	0.66	0.72	0.78	0.83	0.88	0.92	0.96	0.99	1.03
32	0.56	0.64	0.70	0.76	0.80	0.85	0.90	0.93	0.97	1.00
33	0.54	0.61	0.67	0.73	0.78	0.83	0.87	0.91	0.95	0.98
34	0.51	0.59	0.65	0.71	0.76	0.81	0.85	0.89	0.93	0.96
35	0.49	0.56	0.63	0.69	0.74	0.79	0.83	0.87	0.91	0.94
36	0.46	0.54	0.60	0.66	0.72	0.77	0.81	0.85	0.89	0.92
37	0.43	0.51	0.58	0.64	0.70	0.74	0.79	0.83	0.87	0.91
38	0.41	0.49	0.56	0.62	0.67	0.72	0.77	0.81	0.85	0.89
39	0.39	0.47	0.54	0.60	0.65	0.70	0.75	0.79	0.83	0.87
40	0.36	0.44	0.51	0.58	0.63	0.68	0.73	0.77	0.81	0.85
41	0.34	0.42	0.49	0.55	0.61	0.66	0.71	0.75	0.79	0.83
42	0.31	0.40	0.47	0.53	0.59	0.65	0.69	0.74	0.78	0.82
43	0.29	0.37	0.45	0.51	0.57	0.63	0.68	0.72	0.76	0.80
44	0.27	0.35	0.43	0.49	0.55	0.61	0.66	0.70	0.74	0.78
45	0.24	0.33	0.41	0.47	0.53	0.59	0.64	0.68	0.73	0.76
46	0.22	0.31	0.39	0.45	0.51	0.57	0.62	0.67	0.71	0.75
47	0.19	0.29	0.36	0.43	0.50	0.55	0.60	0.65	0.69	0.73
48	0.17	0.26	0.34	0.41	0.48	0.53	0.58	0.63	0.67	0.71
49	0.15	0.24	0.32	0.40	0.46	0.52	0.57	0.61	0.66	0.70
50	0.13	0.22	0.30	0.38	0.44	0.50	0.55	0.60	0.64	0.68
51	0.10	0.20	0.28	0.36	0.42	0.48	0.53	0.58	0.63	0.67
52	0.08	0.18	0.26	0.34	0.40	0.46	0.52	0.56	0.61	0.65
53	0.06	0.16	0.24	0.32	0.39	0.45	0.50	0.55	0.59	0.64
54	0.03	0.14	0.22	0.30	0.37	0.43	0.48	0.53	0.58	0.62
55	0.01	0.12	0.20	0.28	0.35	0.41	0.47	0.52	0.56	0.60
56		0.09	0.18	0.26	0.33	0.39	0.45	0.50	0.55	0.59
57		0.07	0.16	0.24	0.31	0.38	0.43	0.48	0.53	0.58
58		0.05	0.14	0.23	0.30	0.36	0.42	0.47	0.52	0.56
59		0.03	0.13	0.21	0.28	0.34	0.40	0.45	0.50	0.55
60		0.01	0.11	0.19	0.26	0.33	0.39	0.44	0.49	0.53
61			0.09	0.17	0.24	0.31	0.37	0.42	0.47	0.52
62			0.07	0.15	0.23	0.29	0.35	0.41	0.46	0.50
63			0.05	0.13	0.21	0.28	0.34	0.39	0.44	0.49
64			0.03	0.12	0.19	0.26	0.32	0.38	0.43	0.47
65			0.01	0.10	0.18	0.24	0.31	0.36	0.41	0.46
66				0.08	0.16	0.23	0.29	0.35	0.40	0.45
67				0.06	0.14	0.21	0.28	0.33	0.39	0.43
68				0.05	0.13	0.20	0.26	0.32	0.37	0.42
69				0.03	0.11	0.18	0.25	0.30	0.36	0.41
70				0.01	0.09	0.17	0.23	0.29	0.34	0.39
71					0.07	0.15	0.22	0.27	0.33	0.38
72					0.06	0.13	0.20	0.26	0.32	0.37
73					0.04	0.12	0.19	0.25	0.30	0.35
74					0.02	0.10	0.17	0.23	0.29	0.34
75					0.01	0.09	0.16	0.22	0.27	0.33
76						0.07	0.14	0.20	0.26	0.31
77						0.05	0.13	0.19	0.25	0.30
78						0.04	0.11	0.17	0.23	0.29
79						0.02	0.10	0.16	0.22	0.27
80						0.01	0.08	0.15	0.21	0.26
81							0.07	0.13	0.19	0.25
82							0.05	0.12	0.18	0.23
83							0.04	0.10	0.17	0.22
84							0.02	0.09	0.15	0.21
85							0.01	0.08	0.14	0.20
86								0.06	0.13	0.18
87								0.05	0.11	0.17
88								0.03	0.10	0.16
89								0.02	0.09	0.15
90								0.01	0.07	0.13
91									0.06	0.12
92									0.05	0.11
93									0.03	0.09
94									0.02	0.08
95									0.01	0.07
96										0.06
97										0.04
98										0.03
99										0.02
100										0.01

(1) Linear distance on normal probability grid.

NOTES: k values for lower half of series are negative and equal in magnitude to above values if these are numbered in reverse order starting with N. N is number of years of record.

PLOTTING POSITIONS IN PERCENT

(EXCEEDENCE FREQUENCY IN EVENTS PER HUNDRED YEARS)

(See par. 3-05)

m \ N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	N / m
	50.0	29.3	20.6	15.9	13.0	10.9	9.43	8.30	7.43	6.70	6.11	5.62	5.20	4.83	4.52	4.25	4.00	3.78	3.58	3.41	3.25	3.10	1
		70.7	50.0	38.6	31.5	26.5	22.9	20.2	18.1	16.3	14.9	13.7	12.7	11.8	11.0	10.3	9.8	9.2	8.8	8.3	7.9	7.6	2
			79.4	61.4	50.0	42.2	36.5	32.1	28.7	25.9	23.7	21.8	20.1	18.7	17.5	16.4	15.5	14.7	13.9	13.2	12.6	12.0	3
44	98.4			84.1	68.5	57.8	50.0	44.0	39.4	35.6	32.4	29.8	27.6	25.7	24.0	22.5	21.2	20.1	19.1	18.1	17.3	16.5	4
43	96.2	98.4			87.0	73.5	63.5	56.0	50.0	45.2	41.2	37.9	35.1	32.6	30.5	28.6	27.0	25.5	24.2	23.0	22.0	21.0	5
42	93.9	96.1	98.4			89.1	77.1	67.9	60.6	54.8	50.0	46.0	42.5	39.6	37.0	34.7	32.8	31.0	29.4	27.9	26.6	25.4	6
41	91.7	93.8	96.0	98.3			90.6	79.8	71.3	64.4	58.8	54.0	50.0	46.5	43.5	40.8	38.5	36.4	34.5	32.8	31.3	29.9	7
40	89.4	91.5	93.6	95.9	98.3			91.7	81.9	74.1	67.6	62.1	57.5	53.5	50.0	46.9	44.2	41.8	39.7	37.7	36.0	34.4	8
39	87.2	89.2	91.3	93.5	95.8	98.2			92.6	83.7	76.3	70.2	64.9	60.4	56.5	53.1	50.0	47.3	44.8	42.6	40.6	38.8	9
38	84.9	86.9	88.9	91.1	93.3	95.7	98.2			93.3	85.1	78.2	72.4	67.4	63.0	59.2	55.8	52.7	50.0	47.5	45.3	43.3	10
37	82.7	84.6	86.6	88.7	90.8	93.2	95.6	98.1			93.9	86.3	79.9	74.3	69.5	65.3	61.5	58.2	55.2	52.5	50.0	47.8	11
36	80.4	82.3	84.2	86.2	88.4	90.6	93.0	95.5	98.1			94.4	87.3	81.3	76.0	71.4	67.2	63.6	60.3	57.4	54.7	52.2	12
35	78.2	80.0	81.9	83.8	85.9	88.1	90.4	92.8	95.3	98.0			94.8	88.2	82.5	77.5	73.0	69.0	65.5	62.3	59.4	56.7	13
34	75.9	77.4	79.5	81.4	83.4	85.5	87.8	90.1	92.6	95.2	98.0			95.2	89.0	83.6	78.8	74.5	70.6	67.2	64.0	61.2	14
33	73.7	75.4	77.1	79.0	81.0	83.0	85.2	87.4	89.8	92.4	95.1	97.9			95.5	89.7	84.5	79.9	75.8	72.1	68.7	65.6	15
32	71.4	73.0	74.8	76.6	78.5	80.5	82.6	84.8	87.1	89.6	92.2	94.9	97.9			95.8	90.2	85.3	80.9	77.0	73.4	70.1	16
31	69.2	70.7	72.4	74.2	76.0	77.9	80.0	82.1	84.3	86.7	89.3	91.9	94.8	97.8			96.0	90.8	86.1	81.9	78.0	74.6	17
30	66.9	68.4	70.1	71.7	73.5	75.4	77.4	79.4	81.6	83.9	86.3	88.9	91.7	94.6	97.7			96.2	91.2	86.8	82.7	79.0	18
29	64.6	66.1	67.7	69.3	71.0	72.9	74.7	76.7	78.9	81.1	83.4	85.9	88.6	91.4	94.4	97.6			96.4	91.7	87.4	83.5	19
28	62.4	63.8	65.3	66.9	68.6	70.3	72.1	74.1	76.1	78.3	80.5	82.9	85.5	88.2	91.1	94.2	97.6			96.6	92.1	88.0	20
27	60.1	61.5	63.0	64.5	66.1	67.8	69.5	71.4	73.4	75.4	77.6	79.9	82.4	85.0	87.8	90.8	94.1	97.5			96.8	92.4	21
26	57.9	59.2	60.6	62.1	63.6	65.2	66.9	68.7	70.6	72.6	74.7	77.0	79.3	81.9	84.6	87.4	90.5	93.8	97.4			96.9	22
25	55.6	56.9	58.3	59.7	61.1	62.7	64.3	66.0	67.9	69.8	71.8	74.0	76.2	78.7	81.3	84.0	87.0	90.2	93.6	97.3			
24	53.4	54.6	55.9	57.2	58.7	60.2	61.7	63.4	65.1	67.0	68.9	71.0	73.2	75.5	78.0	80.6	83.5	86.5	89.8	93.3	97.2		
23	51.1	52.3	53.5	54.8	56.2	57.6	59.1	60.7	62.4	64.1	66.0	68.0	70.1	72.3	74.7	77.2	80.0	82.9	86.0	89.4	93.0	97.0	23
22	48.9	50.0	51.2	52.4	53.7	55.1	56.5	58.0	59.6	61.3	63.1	65.0	67.0	69.1	71.4	73.8	76.4	79.2	82.2	85.5	89.0	92.8	22
21	46.6	47.7	48.8	50.0	51.2	52.5	53.9	55.3	56.9	58.5	60.2	62.0	63.9	65.9	68.1	70.4	72.9	75.6	78.4	81.5	84.8	88.5	21
20	44.4	45.4	46.5	47.6	48.8	50.0	51.3	52.7	54.1	55.7	57.3	59.0	60.8	62.7	64.8	67.0	69.4	71.9	74.6	77.6	80.8	84.2	20
19	42.1	43.1	44.1	45.2	46.3	47.5	48.7	50.0	51.4	52.8	54.4	56.0	57.9	59.6	61.5	63.6	65.9	68.3	70.8	73.6	76.6	79.9	19
18	39.9	40.8	41.7	42.8	43.8	44.9	46.1	47.3	48.6	50.0	51.5	53.0	54.6	56.4	58.2	60.2	62.3	64.6	67.1	69.7	72.6	75.7	18
17	37.6	38.5	39.4	40.3	41.3	42.4	43.5	44.7	45.9	47.2	48.5	50.0	51.5	53.2	54.9	56.8	58.8	61.0	63.3	65.8	68.4	71.4	17
16	35.4	36.2	37.0	37.9	38.9	39.8	40.9	42.0	43.1	44.3	45.6	47.0	48.5	50.0	51.6	53.4	55.3	57.3	59.5	61.8	64.4	67.1	16
15	33.1	33.9	34.7	35.5	36.4	37.3	38.3	39.3	40.4	41.5	42.7	44.0	45.4	46.8	48.4	50.0	51.8	53.7	55.7	57.9	60.2	62.8	15
14	30.8	31.6	32.3	33.1	33.9	34.8	35.7	36.6	37.6	38.7	39.8	41.0	42.1	43.6	45.1	46.6	48.2	50.0	51.9	53.9	56.2	58.6	14
13	28.6	29.3	29.9	30.7	31.4	32.2	33.1	34.0	34.9	35.9	36.9	38.0	39.2	40.4	41.8	43.2	44.7	46.3	48.1	50.0	52.0	54.3	13
12	26.3	27.0	27.6	28.3	29.0	29.7	30.5	31.3	32.1	33.0	34.0	35.0	36.1	37.3	38.5	39.8	41.2	42.7	44.3	46.1	48.0	50.0	12
11	24.1	24.6	25.2	25.8	26.5	27.1	27.9	28.6	29.4	30.2	31.1	32.0	33.0	34.1	35.2	36.4	37.7	39.0	40.5	42.1	43.8	45.7	11
10	21.8	22.3	22.9	23.4	24.0	24.6	25.3	25.9	26.6	27.4	28.2	29.0	29.9	30.9	31.9	33.0	34.1	35.4	36.7	38.2	39.8	41.4	10
9	19.6	20.0	20.5	21.0	21.5	22.1	22.6	23.3	23.9	24.6	25.3	26.0	26.8	27.7	28.6	29.6	30.6	31.7	32.9	34.2	35.6	37.2	9
8	17.3	17.7	18.1	18.6	19.0	19.5	20.0	20.6	21.1	21.7	22.4	23.0	23.8	24.5	25.3	26.2	27.1	28.1	29.2	30.3	31.6	32.9	8
7	15.1	15.4	15.8	16.2	16.6	17.0	17.4	17.9	18.4	18.9	19.5	20.1	20.7	21.3	22.0	22.8	23.6	24.4	25.4	26.4	27.4	28.6	7
6	12.8	13.1	13.4	13.8	14.1	14.5	14.8	15.2	15.7	16.1	16.6	17.1	17.6	18.1	18.7	19.4	20.0	20.8	21.6	22.4	23.4	24.3	6
5	10.6	10.8	11.1	11.3	11.6	11.9	12.2	12.6	12.9	13.3	13.7	14.1	14.5	15.0	15.4	16.0	16.5	17.1	17.8	18.5	19.2	20.1	5
4	8.3	8.5	8.7	8.9	9.2	9.4	9.6	9.9	10.2	10.4	10.7	11.1	11.4	11.8	12.2	12.6	13.0	13.5	14.0	14.5	15.2	15.8	4
3	6.1	6.2	6.4	6.5	6.7	6.8	7.0	7.2	7.4	7.6	7.8	8.1	8.3	8.6	8.9	9.2	9.5	9.8	10.2	10.6	11.0	11.5	3
2	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.7	4.8	4.9	5.1	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.7	7.0	7.2	2
1	1.56	1.60	1.64	1.68	1.72	1.76	1.81	1.86	1.91	1.96	2.02	2.08	2.15	2.22	2.29	2.36	2.45	2.54	2.64	2.74	2.85	2.97	1
m / N	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	N \ m

PLOTTING POSITIONS IN PERCENT
(EXCEEDENCE FREQUENCY IN EVENTS PER HUNDRED YEARS)

(See par. 3-05)

m \ N	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	N \ m
1	1.53	1.50	1.47	1.43	1.41	1.38	1.35	1.32	1.30	1.28	1.25	1.23	1.21	1.19	1.17	1.15	1.1	1.1	1.1	1.1	1.1	1.0	1
2	3.7	3.7	3.6	3.5	3.4	3.4	3.3	3.2	3.2	3.1	3.1	3.0	3.0	2.9	2.8	2.8	2.7	2.7	2.7	2.6	2.6	2.5	2
3	5.9	5.8	5.7	5.6	5.4	5.3	5.2	5.1	5.0	5.0	4.9	4.8	4.7	4.6	4.5	4.5	4.4	4.3	4.2	4.2	4.1	4.1	3
4	8.1	8.0	7.8	7.6	7.5	7.3	7.2	7.0	6.9	6.8	6.7	6.6	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.6	4
5	10.3	10.1	9.9	9.7	9.5	9.3	9.1	9.0	8.8	8.6	8.5	8.3	8.2	8.0	7.9	7.8	7.6	7.5	7.4	7.3	7.2	7.1	5
6	12.5	12.3	12.0	11.8	11.5	11.3	11.1	10.9	10.7	10.5	10.3	10.1	9.9	9.8	9.6	9.4	9.3	9.1	9.0	8.8	8.7	8.6	6
7	14.7	14.4	14.1	13.8	13.6	13.3	13.1	12.8	12.5	12.3	12.1	11.9	11.7	11.5	11.3	11.1	10.9	10.7	10.6	10.4	10.2	10.1	7
8	17.0	16.7	16.2	15.9	15.6	15.3	15.0	14.7	14.4	14.1	13.9	13.6	13.4	13.2	13.0	12.7	12.5	12.3	12.1	12.0	11.8	11.6	8
9	19.2	18.7	18.3	18.0	17.6	17.2	16.9	16.6	16.3	16.0	15.7	15.4	15.2	14.9	14.6	14.4	14.2	13.9	13.7	13.5	13.3	13.1	9
10	21.4	20.9	20.5	20.0	19.6	19.2	18.9	18.5	18.2	17.8	17.5	17.2	16.9	16.6	16.3	16.1	15.8	15.5	15.3	15.1	14.8	14.6	10
11	23.6	23.1	22.6	22.1	21.6	21.2	20.8	20.4	20.0	19.7	19.3	19.0	18.6	18.3	18.0	17.7	17.4	17.1	16.9	16.6	16.4	16.1	11
12	25.8	25.2	24.7	24.2	23.7	23.2	22.8	22.3	21.9	21.5	21.1	20.7	20.4	20.0	19.7	19.4	19.0	18.7	18.4	18.2	17.9	17.6	12
13	28.0	27.4	26.8	26.2	25.7	25.2	24.7	24.2	23.8	23.3	22.9	22.5	22.1	21.7	21.4	21.0	20.7	20.3	20.0	19.7	19.4	19.1	13
14	30.2	29.5	28.9	28.3	27.7	27.2	26.6	26.1	25.7	25.2	24.7	24.3	23.9	23.5	23.1	22.7	22.3	21.9	21.6	21.3	20.9	20.6	14
15	32.4	31.7	31.0	30.4	29.8	29.2	28.6	28.0	27.5	27.0	26.5	26.1	25.6	25.2	24.7	24.3	23.9	23.6	23.2	22.8	22.5	22.1	15
16	34.6	33.8	33.1	32.4	31.8	31.1	30.5	30.0	29.4	28.9	28.3	27.8	27.3	26.9	26.4	26.0	25.6	25.2	24.8	24.4	24.0	23.6	16
17	36.8	36.0	35.2	34.5	33.8	33.1	32.5	31.9	31.3	30.7	30.1	29.6	29.1	28.6	28.1	27.6	27.2	26.8	26.3	25.9	25.5	25.1	17
18	39.0	38.1	37.3	36.6	35.8	35.1	34.4	33.8	33.1	32.5	31.9	31.4	30.8	30.3	29.8	29.3	28.8	28.4	27.9	27.5	27.1	26.7	18
19	41.2	40.3	39.4	38.6	37.8	37.1	36.4	35.7	35.0	34.4	33.8	33.2	32.6	32.0	31.5	31.0	30.5	30.0	29.5	29.0	28.6	28.2	19
20	43.4	42.5	41.6	40.7	39.9	39.1	38.3	37.6	36.9	36.2	35.6	34.9	34.3	33.7	33.2	32.6	32.1	31.6	31.1	30.6	30.1	29.7	20
21	45.6	44.6	43.7	42.8	41.9	41.1	40.3	39.5	38.8	38.0	37.4	36.7	36.1	35.4	34.8	34.3	33.7	33.2	32.6	32.1	31.6	31.2	21
22	47.8	46.8	45.8	44.8	43.9	43.1	42.2	41.4	40.6	39.9	39.2	38.5	37.8	37.2	36.5	35.9	35.3	34.8	34.2	33.7	33.2	32.7	22
23	50.0	48.9	47.9	46.9	46.0	45.0	44.2	43.3	42.5	41.7	41.0	40.2	39.5	38.9	38.2	37.6	37.0	36.4	35.8	35.2	34.7	34.2	23
			50.0	49.0	48.0	47.0	46.1	45.2	44.4	43.6	42.8	42.0	41.3	40.6	39.9	39.2	38.6	38.0	37.4	36.8	36.2	35.7	24
					50.0	49.0	48.1	47.1	46.3	45.4	44.6	43.8	43.0	42.3	41.6	40.9	40.2	39.6	39.0	38.4	37.8	37.2	25
							50.0	49.0	48.1	47.2	46.4	45.6	44.8	44.0	43.3	42.5	41.9	41.2	40.5	39.9	39.3	38.7	26
									50.0	49.1	48.2	47.3	46.5	45.7	44.9	44.2	43.5	42.8	42.1	41.5	40.8	40.2	27
											50.0	49.1	48.3	47.4	46.6	45.9	45.1	44.4	43.7	43.0	42.4	41.7	28
													50.0	49.1	48.3	47.5	46.7	46.0	45.3	44.6	43.9	43.2	29
															50.0	49.2	48.4	47.6	46.9	46.1	45.4	44.7	30
																	50.0	49.2	48.4	47.7	46.9	46.2	31
																			50.0	49.2	48.5	47.7	32
																					50.0	49.2	33

NOTES:

1. Plotting positions are symmetrical about 50 percent.

2. For arrays exceeding 66 values, plotting positions can readily be obtained by use of desk calculator and constants given below.

N	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	N
P_1	1.03	1.01	1.00	.99	.97	.96	.94	.93	.92	.91	.90	.88	.87	.86	.85	.84	.83	P_1
ΔP	1.4839	1.4624	1.4412	1.4206	1.4008	1.3814	1.3628	1.3444	1.3265	1.3091	1.2921	1.2758	1.2597	1.2440	1.2288	1.2138	1.1993	ΔP

N	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	N
P_1	.82	.81	.80	.79	.78	.78	.77	.76	.75	.74	.73	.73	.72	.71	.71	.70	.69	P_1
ΔP	1.1851	1.1712	1.1576	1.1444	1.1315	1.1186	1.1063	1.0942	1.0824	1.0709	1.0596	1.0483	1.0375	1.0269	1.0163	1.0061	.9962	ΔP

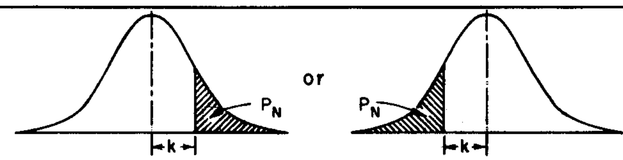
For computer application, the formula (m-.3)/(N+.4) is used.

TABLE OF k VERSUS P_N

For use with samples drawn from a normal population. (See par. 4-03d)

N-1 \ P_N (%)	50.0	45.0	40.0	35.0	30.0	25.0	20.0	15.0	12.5	10.0	5.0	2.5	1.25	1.0	0.5	0.25	0.05
1	.00	.19	.40	.62	.89	1.22	1.69	2.40	2.96	3.77	7.73	15.56	31.17	38.97	77.96	155.93	779.70
2	.00	.16	.33	.51	.71	.94	1.23	1.60	1.85	2.18	3.37	4.97	7.16	8.04	11.46	16.27	36.49
3	.00	.15	.31	.47	.65	.86	1.09	1.40	1.59	1.83	2.63	3.56	4.67	5.08	6.53	8.33	14.47
4	.00	.15	.30	.45	.62	.81	1.03	1.30	1.47	1.68	2.34	3.04	3.83	4.10	5.04	6.13	9.43
5	.00	.14	.29	.44	.60	.79	.99	1.25	1.41	1.59	2.18	2.78	3.42	3.63	4.36	5.16	7.41
6	.00	.14	.28	.43	.59	.77	.97	1.21	1.36	1.54	2.08	2.62	3.17	3.36	3.96	4.62	6.37
7	.00	.14	.28	.43	.58	.75	.95	1.19	1.33	1.50	2.01	2.51	3.01	3.18	3.71	4.27	5.73
8	.00	.14	.28	.42	.58	.74	.94	1.17	1.31	1.47	1.96	2.43	2.90	3.05	3.54	4.04	5.31
9	.00	.14	.27	.42	.57	.74	.93	1.15	1.29	1.45	1.92	2.37	2.82	2.96	3.41	3.87	5.01
10	.00	.13	.27	.41	.57	.73	.92	1.14	1.28	1.43	1.89	2.33	2.75	2.89	3.31	3.74	4.79
11	.00	.13	.27	.41	.56	.73	.91	1.13	1.26	1.42	1.87	2.29	2.70	2.83	3.23	3.64	4.62
12	.00	.13	.27	.41	.56	.72	.91	1.12	1.25	1.41	1.85	2.26	2.66	2.78	3.17	3.56	4.48
13	.00	.13	.27	.41	.56	.72	.90	1.12	1.25	1.40	1.83	2.24	2.62	2.74	3.12	3.49	4.37
14	.00	.13	.27	.41	.55	.71	.90	1.11	1.24	1.39	1.82	2.22	2.59	2.71	3.07	3.44	4.28
15	.00	.13	.27	.41	.55	.71	.89	1.11	1.23	1.38	1.81	2.20	2.57	2.68	3.04	3.39	4.20
16	.00	.13	.27	.40	.55	.71	.89	1.10	1.23	1.38	1.80	2.18	2.54	2.66	3.01	3.35	4.13
17	.00	.13	.26	.40	.55	.71	.89	1.10	1.22	1.37	1.79	2.17	2.53	2.64	2.98	3.31	4.07
18	.00	.13	.26	.40	.55	.71	.88	1.09	1.22	1.36	1.78	2.16	2.51	2.62	2.95	3.28	4.02
19	.00	.13	.26	.40	.55	.70	.88	1.09	1.22	1.36	1.77	2.14	2.49	2.60	2.93	3.25	3.98
20	.00	.13	.26	.40	.55	.70	.88	1.09	1.21	1.36	1.77	2.14	2.48	2.59	2.91	3.23	3.94
21	.00	.13	.26	.40	.54	.70	.88	1.09	1.21	1.35	1.76	2.13	2.47	2.57	2.89	3.21	3.90
22	.00	.13	.26	.40	.54	.70	.88	1.08	1.21	1.35	1.75	2.12	2.46	2.56	2.88	3.19	3.87
23	.00	.13	.26	.40	.54	.70	.88	1.08	1.20	1.35	1.75	2.11	2.45	2.55	2.86	3.17	3.84
24	.00	.13	.26	.40	.54	.70	.87	1.08	1.20	1.34	1.74	2.10	2.44	2.54	2.85	3.15	3.82
25	.00	.13	.26	.40	.54	.70	.87	1.08	1.20	1.34	1.74	2.10	2.43	2.53	2.84	3.14	3.80
26	.00	.13	.26	.40	.54	.70	.87	1.08	1.20	1.34	1.74	2.09	2.42	2.52	2.83	3.12	3.78
27	.00	.13	.26	.40	.54	.70	.87	1.08	1.20	1.34	1.73	2.09	2.42	2.52	2.82	3.11	3.76
28	.00	.13	.26	.40	.54	.69	.87	1.07	1.20	1.34	1.73	2.08	2.41	2.51	2.81	3.10	3.74
29	.00	.13	.26	.40	.54	.69	.87	1.07	1.19	1.33	1.73	2.08	2.40	2.50	2.80	3.09	3.72
30	.00	.13	.26	.40	.54	.69	.87	1.07	1.19	1.33	1.72	2.07	2.40	2.50	2.79	3.08	3.70
40	.00	.13	.26	.39	.54	.69	.86	1.06	1.18	1.32	1.70	2.05	2.36	2.45	2.74	3.01	3.59
60	.00	.13	.26	.39	.53	.68	.85	1.05	1.17	1.31	1.68	2.02	2.32	2.41	2.68	2.94	3.49
120	.00	.13	.26	.39	.53	.68	.85	1.05	1.16	1.29	1.66	1.99	2.28	2.37	2.63	2.87	3.39
∞	.00	.13	.25	.38	.52	.67	.84	1.04	1.15	1.28	1.64	1.96	2.24	2.33	2.58	2.81	3.29

This table shows k for values of P_N as illustrated:



PEARSON TYPE III COORDINATES

(See par. 4-03 d)

g (Skew coefficient)	k = Magnitude in standard deviations from mean for exceedence percentages of: 0.01	0.1	1.0	5	10	30	50	70	90	95	99	99.9	99.99
1.0	5.92	4.54	3.03	1.87	1.34	0.38	-0.16	-0.61	-1.12	-1.31	-1.59	-1.80	-1.88
0.8	5.48	4.25	2.90	1.83	1.34	0.42	-0.13	-0.60	-1.16	-1.38	-1.74	-2.03	-2.18
0.6	5.04	3.96	2.77	1.79	1.33	0.45	-0.09	-0.58	-1.19	-1.45	-1.88	-2.28	-2.53
0.4	4.60	3.67	2.62	1.74	1.32	0.48	-0.06	-0.57	-1.22	-1.51	-2.03	-2.54	-2.92
0.2	4.16	3.38	2.48	1.69	1.30	0.51	-0.03	-0.55	-1.25	-1.58	-2.18	-2.81	-3.32
0.0	3.73	3.09	2.33	1.64	1.28	0.52	0.00	-0.52	-1.28	-1.64	-2.33	-3.09	-3.73
-0.2	3.32	2.81	2.18	1.58	1.25	0.55	0.03	-0.51	-1.30	-1.69	-2.48	-3.38	-4.16
-0.4	2.92	2.54	2.03	1.51	1.22	0.57	0.06	-0.48	-1.32	-1.74	-2.62	-3.67	-4.60
-0.6	2.53	2.28	1.88	1.45	1.19	0.58	0.09	-0.45	-1.33	-1.79	-2.77	-3.96	-5.04
-0.8	2.18	2.03	1.74	1.38	1.16	0.60	0.13	-0.42	-1.34	-1.83	-2.90	-4.25	-5.48
-1.0	1.88	1.80	1.59	1.31	1.12	0.61	0.16	-0.38	-1.34	-1.87	-3.03	-4.54	-5.92
Skew Coefficients Commonly Used													
.00	3.73	3.09	2.33	1.64	1.28	0.52	0.00	-0.52	-1.28	-1.64	-2.33	-3.09	-3.73
-.04	3.65	3.03	2.30	1.63	1.27	0.53	0.01	-0.52	-1.28	-1.65	-2.36	-3.15	-3.82
-.12	3.48	2.92	2.24	1.60	1.26	0.54	0.02	-0.51	-1.29	-1.67	-2.42	-3.26	-3.99
-.23	3.26	2.77	2.16	1.57	1.25	0.55	0.03	-0.50	-1.30	-1.70	-2.50	-3.42	-4.23
-.32	3.08	2.68	2.09	1.54	1.23	0.56	0.05	-0.49	-1.31	-1.72	-2.56	-3.55	-4.42
-.37	2.98	2.58	2.05	1.52	1.22	0.57	0.06	-0.48	-1.32	-1.73	-2.60	-3.63	-4.53
-.40	2.92	2.54	2.03	1.51	1.22	0.57	0.06	-0.48	-1.32	-1.74	-2.62	-3.67	-4.60

NOTE: Approximate transformations between normal deviate (X) and Pearson Type III deviate k can be accomplished with the following equation:

$$k = \frac{2}{g}\left\{\left[\frac{g}{6}\left(X - \frac{g}{6}\right) + 1\right]^{3} - 1\right\}$$

TABLE OF P_N VERSUS P_∞ IN PERCENT

For use with samples drawn from a normal population
(See par. 4-03 d)

N-1 \ P_∞	50.0	30.0	10.0	5.0	1.0	0.1	0.01
1	50.0	37.2	24.3	20.4	15.4	12.1	10.2
2	50.0	34.7	19.3	14.6	9.0	5.7	4.3
3	50.0	33.6	16.9	11.9	6.4	3.5	2.3
4	50.0	33.0	15.4	10.4	5.0	2.4	1.37
5	50.0	32.5	14.6	9.4	4.2	1.79	.92
6	50.0	32.2	13.8	8.8	3.6	1.38	.66
7	50.0	31.9	13.5	8.3	3.2	1.13	.50
8	50.0	31.7	13.1	7.9	2.9	.94	.39
9	50.0	31.6	12.7	7.6	2.7	.82	.31
10	50.0	31.5	12.5	7.3	2.5	.72	.25
11	50.0	31.4	12.3	7.1	2.3	.64	.21
12	50.0	31.3	12.1	6.9	2.2	.58	.18
13	50.0	31.2	11.9	6.8	2.1	.52	.16
14	50.0	31.1	11.8	6.7	2.0	.48	.14
15	50.0	31.1	11.7	6.6	1.96	.45	.13
16	50.0	31.0	11.6	6.5	1.90	.42	.12
17	50.0	31.0	11.5	6.4	1.84	.40	.11
18	50.0	30.9	11.4	6.3	1.79	.38	.10
19	50.0	30.9	11.3	6.2	1.74	.36	.091
20	50.0	30.8	11.3	6.2	1.70	.34	.084
21	50.0	30.8	11.2	6.1	1.67	.33	.078
22	50.0	30.8	11.1	6.1	1.63	.31	.073
23	50.0	30.7	11.1	6.0	1.61	.30	.068
24	50.0	30.7	11.0	6.0	1.58	.29	.064
25	50.0	30.7	11.0	5.9	1.55	.28	.060
26	50.0	30.6	10.9	5.9	1.53	.27	.057
27	50.0	30.6	10.9	5.9	1.51	.26	.054
28	50.0	30.6	10.9	5.8	1.49	.26	.051
29	50.0	30.6	10.8	5.8	1.47	.25	.049
30	50.0	30.6	10.8	5.8	1.45	.24	.046
40	50.0	30.4	10.6	5.6	1.33	.20	.034
60	50.0	30.3	10.4	5.4	1.22	.16	.025
120	50.0	30.2	10.2	5.2	1.11	.13	.017
∞	50.0	30.0	10.0	5.0	1.00	.10	.010

NOTE: P_N values above are usable approximately with Pearson Type III distributions having small skew coefficients.

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9
10	000	004	009	013	017	021	025	029	033	037
11	041	045	049	053	057	061	064	068	072	076
12	079	083	086	090	093	097	100	104	107	111
13	114	117	121	124	127	130	134	137	140	143
14	146	149	152	155	158	161	164	167	170	173
15	176	179	182	185	188	190	193	196	199	201
16	204	207	210	212	215	217	220	223	225	228
17	230	233	236	238	241	243	246	248	250	253
18	255	258	260	262	265	267	270	272	274	276
19	279	281	283	286	288	290	292	294	297	299
20	301	303	305	308	310	312	314	316	318	320
21	322	324	326	328	330	332	334	336	338	340
22	342	344	346	348	350	352	354	356	358	360
23	362	364	366	367	369	371	373	375	377	378
24	380	382	384	386	387	389	391	393	394	396
25	398	400	401	403	405	407	408	410	412	413
26	415	417	418	420	422	423	425	427	428	430
27	431	433	435	436	438	439	441	442	444	446
28	447	449	450	452	453	455	456	458	459	461
29	462	464	465	467	468	470	471	473	474	476
30	477	479	480	481	483	484	486	487	489	490
31	491	493	494	496	497	498	500	501	502	504
32	505	507	508	509	511	512	513	515	516	517
33	519	520	521	522	524	525	526	528	529	530
34	531	533	534	535	537	538	539	540	542	543
35	544	545	547	548	549	550	551	553	554	555
36	556	558	559	560	561	562	563	565	566	567
37	568	569	571	572	573	574	575	576	577	579
38	580	581	582	583	584	585	587	588	589	590
39	591	592	593	594	596	597	598	599	600	601
40	602	603	604	605	606	607	609	610	611	612
41	613	614	615	616	617	618	619	620	621	622
42	623	624	625	626	627	628	629	630	631	632
43	633	634	635	636	637	638	639	640	641	642
44	643	644	645	646	647	648	649	650	651	652
45	653	654	655	656	657	658	659	660	661	662
46	663	664	665	666	667	667	668	669	670	671
47	672	673	674	675	676	677	678	679	679	680
48	681	682	683	684	685	686	687	688	688	689
49	690	691	692	693	694	695	695	696	697	698
50	699	700	701	702	702	703	704	705	706	707
51	708	708	709	710	711	712	713	713	714	715
52	716	717	718	719	719	720	721	722	723	723
53	724	725	726	727	728	728	729	730	731	732
54	732	733	734	735	736	736	737	738	739	740
55	740	741	742	743	744	744	745	746	747	747
56	748	749	750	751	751	752	753	754	754	755
57	756	757	757	758	759	760	760	761	762	763
58	763	764	765	766	766	767	768	769	769	770
59	771	772	772	773	774	775	775	776	777	777
60	778	779	780	780	781	782	782	783	784	785
61	785	786	787	787	788	789	790	790	791	792
62	792	793	794	794	795	796	797	797	798	799
63	799	800	801	801	802	803	803	804	805	806
64	806	807	808	808	809	810	810	811	812	812
65	813	814	814	815	816	816	817	818	818	819
66	820	820	821	822	822	823	823	824	825	825
67	826	827	827	828	829	829	830	831	831	832
68	833	833	834	834	835	836	836	837	838	838
69	839	839	840	841	841	842	843	843	844	844
70	845	846	846	847	848	848	849	849	850	851
71	851	852	852	853	854	854	855	856	856	857
72	857	858	859	859	860	860	861	862	862	863
73	863	864	865	865	866	866	867	867	868	869
74	869	870	870	871	872	872	873	873	874	875
75	875	876	876	877	877	878	879	879	880	880
76	881	881	882	883	883	884	884	885	885	886
77	886	887	888	888	889	889	890	890	891	892
78	892	893	893	894	894	895	895	896	897	897
79	898	898	899	899	900	900	901	902	902	903
80	903	904	904	905	905	906	906	907	907	908
81	908	909	910	910	911	911	912	912	913	913
82	914	914	915	915	916	916	917	918	918	919
83	919	920	920	921	921	922	922	923	923	924
84	924	925	925	926	926	927	927	928	928	929
85	929	930	930	931	931	932	932	933	933	934
86	935	935	936	936	937	937	938	938	939	939
87	940	940	941	941	942	942	943	943	943	944
88	944	945	945	946	946	947	947	948	948	949
89	949	950	950	951	951	952	952	953	953	954
90	954	955	955	956	956	957	957	958	958	959
91	959	960	960	960	961	961	962	962	963	963
92	964	964	965	965	966	966	967	967	968	968
93	968	969	969	970	970	971	971	972	972	973
94	973	974	974	975	975	975	976	976	977	977
95	978	978	979	979	980	980	980	981	981	982
96	982	983	983	984	984	985	985	985	986	986
97	987	987	988	988	989	989	989	990	990	991
98	991	992	992	993	993	993	994	994	995	995
99	996	996	997	997	997	998	998	999	999	999

NOTE: Logarithm of a number is the algebraic sum of above mantissas (preceded by a decimal point) and the characteristic. The characteristic is one less than the number of digits left of the decimal point if number is one or greater. If number is less than 1, characteristic is negative and equal to one more than the number of consecutive ciphers immediately following the decimal point.

Examples

Number	log
4380	3.641
2.1	.322
0.21	-.678
0.04	-1.398

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